Reliability Assessment of Granular Filters in Embankment Dams

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Abstract

Empirical criteria have been used successfully to design filters of most embankment large dam projects throughout the world. However, these empirical rules are only applicable to a particular range of soils tested in laboratory and do not take into account the variability of the base material and filter particle sizes. In addition, it is widely accepted that the safety of fill dams is mainly dependent on the reliability of their filter performance. The work herein presented consists in a new general method for assessing the probability of fulfilling any empirical filter design criteria accounting for base and filter heterogeneity by means of first order reliability methods (FORM), so that reliability indexes and probabilities of fulfilling any particular criteria are obtained. This method will allow engineers to estimate the safety of existing filters in terms of probability of fulfilling their design criteria and might also be used as a decision tool on sampling needs and material size tolerances during construction. In addition, sensitivity analysis makes possible to analyze how reliabilities are influenced by different sources of input data. Finally, in case of a portfolio risk assessment, this method will allow engineers to compare the safety of several existing dams in order to prioritize safety investments and it is expected to be a very useful tool to evaluate probabilities of failure due to internal erosion.

CE Database Subject Headings: Embankment dams; Erosion; Filtration; Granular filters; Level II (FORM) methods; Risk assessment; System reliability.

1 Introduction and Motivation

It is now well known that cracks can develop within a well constructed dam core, leading to concentrated seepage and high erosion rates, which can compromise the safety of the embankment dam. As a matter of fact, internal erosion is one of the most important causes of failure in embankment dams. The annual probability of failure due to internal erosion of a large modern dam during operation is estimated (based on historical data) in 10^{-5}, slightly less than the probability of failure due to overtopping, but well ahead of failure due to sliding (see Fry et al. (1997)).

The sequence of events leading to the failure of a dam by internal erosion with a concentrated leak is described in Figure 1. The best way to prevent internal erosion is using adequate granular filters (or geotextiles) in the transition areas where important hydraulic gradients can appear. In case of cracking and erosion, if the filter is capable of retaining the eroded particles, then the crack will seal and the dam safety will be ensured. Hence, granular filters are one of the most

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important elements in embankment dams. The trust in these filters is such that Sherard and Dunningan (1985) affirmed: *"By providing a conservative downstream filter, we can quit worrying about possible concentrated leaks through the core."*

Filters are mainly designed using simply to apply empirical criteria. These criteria have been proposed by researches as the result of a correlation between different base soil ("Base soil" refers to any material of the dam or foundation that is able to suffer internal erosion and must be protected, for example, the clay core of an embankment dam) and filter variables that produce a satisfactory behavior if they are tested in laboratory under extreme conditions. These empirical criteria indirectly take into account all the factors affecting filtration but are only applicable to the range of soils tested and depend on testing methods, definitions of failure, etc.

Although no dam designed in accordance with modern filter requirements has ever suffered incidents related to internal erosion (see Fry et al. (1997)), if we attempt to design a new dam or to evaluate the safety of a existing dam, it is very difficult to analyze the reliability of the filter because the empirical criteria do not take into account the variability of the base and filter particle size along the filter and the core of the dam, so that it is not possible to establish the real safety level or the probability of failure of the filter. Note that in this paper, the failure of the system filter-base is related to the no satisfaction of the empirical criteria.

In spite of the simplicity and good behavior of dams designed using these empirical criteria, this traditional design procedure presents the following shortcomings:

1. Statistical variability of filter and base size is not taken into account.

2. It is not possible to determine the safety level of the filter. Either it holds the empirical criteria or it does not, but not intermediate situations are possible.

3. It is very difficult to compare the levels of risk between different dams, for example, in order to prioritize the rehabilitation investments.

4. It is very difficult to carry out a sensitivity analysis, which would allow identifying the most important variables to be controlled strictly.

The reliability assessment method proposed in this paper is based on considering 'failure' of the filter-base system the non-fulfillment of the empirical design criteria. For illustration purposes we have selected the most common widely accepted criteria but it is very important to highlight that the method is very flexible and allows an easy modification of the different criteria.

In addition to the reliability assessment, some interest is shown by people in knowing how sensitive are the reliabilities to data values. A sensitivity analysis provides excellent information on the extent to which a small change in the parameters or assumptions (data) modifies the resulting reliabilities.

The aims of this paper are: (a) to present a method for evaluating the safety level related to the different empirical design criteria, and (b) to provide tools to perform a sensitivity analysis.

The paper is structured as follows. In Section 2 the proposed method for reliability assessment is presented. In Section 4 a technique for performing a sensitivity analysis is explained. Section 3 shows the analytical solution for certain kind of problems using the proposed method. Section 5 presents the numerical results for some specific base-filter systems. And finally, in Section 6 some conclusions are given.

2 Proposed Method for Reliability Assessment

The reliability analysis applied to different engineering works (such as dams, breakwaters, slope stabilizations, etc.), assumes that there are some random variables $(X_1, \ldots, X_n)$ involved. However, in the methodology herein proposed, no distinction is made between random and deterministic variables. In consequence, it is assumed that all variables are random, and deterministic variables are only particular cases of them. Upper-case letters are used to refer to random variables, and the corresponding lowercase letters are used to refer to particular instantiations of these variables. They belong to an n-dimensional space, which, for each mode of failure, can be divided into two zones: the so called safe and the failure regions:

$$\left.\begin{array}{ll}\text{Safe Region:} & \mathcal{S} \equiv \{(x_1, x_2, \ldots, x_n)\} | g_m(x_1, x_2, \ldots, x_n) > 0\} \\ \text{Failure Region:} & \mathcal{F} \equiv \{(x_1, x_2, \ldots, x_n)\} | g_m(x_1, x_2, \ldots, x_n) \leq 0\}\end{array}\right\}; \quad m \in M, \tag{1}$$

where M is the set of all modes of failure.

In this paper, the set of basic variables $(X_1, \ldots, X_n)$ will be partitioned in two sets:

1. **Random variables $\boldsymbol{\eta}$:** Their mean or characteristic values are fixed by the engineer or the code guidelines as input data, or they come from existing data samples. They include base and filter particle size variables (D_{iF}, d_{iB}) and the stability rules F_m derived from the experiments, where D_{iF} and d_{iB} are the diameter of filter and base particles, respectively, for which (i)% of the entire mass is finer. The corresponding mean of $\boldsymbol{\eta}$ is denoted $\tilde{\boldsymbol{\eta}}$.

2. **Random model parameters $\boldsymbol{\kappa}$:** Set of parameters defining the random variability and dependence structure of the random variables involved (standard deviations, variation coefficients, correlations, covariance matrices, etc.). In this article the coefficients of variation of the base and filter particle size variables $v_{D_{5F}}, v_{D_{10F}}, \ldots, v_{d_{75B}}$, and the correlation coefficients $\boldsymbol{\rho}$ between filter particle size variables are considered.

The probability of failure P_m under mode m can be calculated using the joint probability density function of all variables involved by means of the integral:

$$P_m(\boldsymbol{\theta}) = \int\limits_{g_m(x_1, x_2, \ldots, x_n) \leq 0} f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n; \boldsymbol{\theta}) dx_1 dx_2 \ldots dx_n, \tag{2}$$

where $\boldsymbol{\theta} = (\tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ is a parametric vector containing the mean values $\tilde{\boldsymbol{\eta}}$, and the vector of random model parameters $\boldsymbol{\kappa}$.

2.1 Modes of failure or Practical Design Criteria

Before performing the reliability assessment, limit state equations associated with the different practical criteria (modes of failure) must be defined. In this study a total of 11 modes of failure have been considered: retention criteria (r), which is composed by two different limit state equations (r_a) and (r_b), permeability criteria (p), filter uniformity criteria (u), six auto-stability criteria $(a_5, a_{15}, a_{30}, a_{50}, a_{70}, a_{85})$, and non-cohesive criteria (c). All modes of failure are ascribed to non desirable situations.

These criteria have been defined by many different researchers as reviewed by Indraratna and Locke (1999), and some specific design criteria have been even recently developed (Delgado (2000)), but only those most generally accepted are considered in this paper.

Retention criterion. 'Filters must be fine enough so that the pore spaces between the filter particles can hold some of the larger particles of the base soil'. The most generally accepted retention criteria was proposed by Sherard and Dunningan (1985) who classified base soils in 4 groups. In this paper group 1 (more than 85% passing 0.075 *mm* sieve) and group 2 (between 40% and 85% passing 0.075 *mm* sieve) base soils are considered.

Retention criterion can be verified by the following limit state equation:

$$\textbf{Group 1:} \quad g_r = 9 - \frac{D_{15F}}{d_{85B}}, \quad \text{if} \quad D_{15F} \geq 0.2\, mm, \tag{3}$$

$$\textbf{Group 2:} \quad g_r = D_{15F} - 0.7, \tag{4}$$

where D_{15F} and d_{85B} are the diameters of filter and base particles for which 15% and 85% of the entire mass is finer, respectively. Note that for group 1 soils no failure occurs if $D_{15F} < 0.2\, mm$, and this fact will affect the evaluation of the reliability associated with this criterion, where system reliability concepts are required. Thus, this failure mode can be considered as a parallel system composed by two components:

$$g_{r_a} = 9 - \frac{D_{15F}}{d_{85B}} \tag{5}$$

$$g_{r_b} = 0.2 - D_{15F}, \tag{6}$$

where the failure of the system requires the failure of both components simultaneously. In the case of group 2 soils, a unique limit state equation, which is not dependent on the variability of the base soil, is required.

Permeability criterion. 'Filter must be coarse enough to allow seepage flow to pass through the filter, preventing build up of high pressures and hydraulic gradients". Filter and soil base permeabilities must be considered directly or by means of correlations with their particle size distributions.

Permeability criterion can be verified by the following limit state equation:

$$g_p = \frac{D_{15F}}{d_{15B}} - 4, \tag{7}$$

where D_{15F} and d_{15B} are the diameters of filter and base particles for which 15% of the entire mass is finer, respectively.

Filter uniformity criterion. Filter uniformity criterion can be verified by the following limit state equation:

$$g_u = 20 - \frac{D_{60F}}{D_{10F}}, \tag{8}$$

where D_{60F} and D_{10F} are the diameters of filter particles for which 60% and 10% of the entire mass is finer, respectively.

Auto-stability criteria. When water flows through the filter, its fine particles should not move within the skeleton of the coarse ones leading to erosion. Auto-stability criteria can be verified by the following limit state equations proposed by Kenney and Lao (1985):

$$g_{a_i} = 5 - \frac{D_{(15+i)F}}{D_{iF}}; \; i \in \{5, 15, 30, 50, 70, 85\}, \tag{9}$$

where $D_{(15+i)F}$ and D_{iF} are the diameters of filter particles for which $(15+i)\%$ and $i\%$ of the entire mass is finer, respectively.

Non-cohesive criterion. If a filter has cohesion it may sustain a crack where base particles can pass through. To ensure that the filter has no cohesion, it should contain no more than 5% fines passing 0.075 mm sieve and such fines should be non-plastic. Non-cohesive criterion can be verified by the following limit state equation:

$$g_c = D_{5F} - 0.075, \tag{10}$$

where D_{5F} is the diameter of filter particle for which 5% of the entire mass is finer.

Note that failure for each individual criterion is considered when the corresponding limit state equations (5)-(10) are lower than zero, respectively.

2.2 Evaluation of the failure mode probabilities

In this work the probabilities associated with each failure mode are evaluated using "First Order Reliability Methods" (FORM) (Freudenthal (1956); Hasofer and Lind (1974); Rackwitz and Fiessler (1978); Hohenbichler and Rackwitz (1981); Ditlevsen (1981)). This methodology gives precise results (Madsen et al. (1986), Ditlevsen and Madsen (1996), or Melchers (1999)) and is much more efficient than Monte Carlo simulation techniques for estimating extreme percentiles (Wirsching and Wu (1987), or Haskin et al. (1996)). More precisely, $P_m(\boldsymbol{\theta})$ for $m = 1, 2, \ldots, M$ is obtained by means of the reliability index using:

$$\beta_m(\boldsymbol{\theta}) = \underset{\boldsymbol{\eta}}{\text{Minimum}} \quad \sqrt{\boldsymbol{z}^T\boldsymbol{z}} \ , \tag{11}$$

i.e., minimizing with respect to $\boldsymbol{\eta}$, subject to

$$\begin{aligned} \boldsymbol{z} &= T(\boldsymbol{\eta}, \boldsymbol{\theta}) && (12)\\ g_m(\boldsymbol{\eta}) &= 0, && (13) \end{aligned}$$

where β_m is the reliability index for failure mode $m \in M$, $T(\boldsymbol{\eta}, \boldsymbol{\theta})$ is the transformation (Rosenblatt (1952), Nataf (1962)) leading to the standard unit normal $\boldsymbol{z}$ variables used in FORM and $g_m(\boldsymbol{\eta}) = 0$ is the boundary of the failure region for failure mode m defined by the practical criterion m.

Note that the problem in (11)–(13) can give the wrong answer, that is, a positive value of β when the correct answer is a negative β. This is due to the fact that two square roots are possible in (11). To get the right sign we add the following constraints:

$$\begin{aligned} \boldsymbol{0} &= T(\boldsymbol{\eta}_1, \boldsymbol{\theta}) && (14)\\ g_m(\boldsymbol{\eta}_1)u_m &> 1, && (15) \end{aligned}$$

where the auxiliary variable u_m and the two constraints (14) and (15) ensure that the sign of β_m is the desired one and $\boldsymbol{\eta}_1$ is the random variable values corresponding to the point $\boldsymbol{z}_1 = \boldsymbol{0}$ in the standard normal random space. Therefore, the final reliability index will be $\beta_m = \text{sign}(u_m)\beta_m$.

The probability of failure P_m is related to the reliability indices by the approximate relation $P_m = \Phi(-\beta_m)$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable. If the failure region is linear in the standard normal random space the probability becomes exact.

As it was mentioned before, the probability of failure obtained using this method is the probability of non-fulfillment of the empirical criteria based on experiments developed under extreme conditions (high hydraulic gradients). If we want to extend the result to the probability of non-fulfillment inside the dam, Bayes theorem should be used $P_{f_m} = P_m P_{ec}$, where P_{f_m} is the probability of non-fulfillment of the empirical criteria m in the dam, P_m is the probability of non-fulfillment of the empirical criteria conditioned to extreme conditions occur (the one calculated in this paper), and P_{ec} is the probability of reaching, within the dam, the same extreme conditions (related to cracking, hydraulic gradients, etc.) as in the experiments. Note that actually there is no method available for calculating exactly this probability.

2.3 System reliability assessment

In the previous section the evaluation of the probability of failure associated with the different failure modes was dealt with, but considering the probability of fulfilling several criteria at the same time 'structural system' reliability assessment methods are required. The following 'structural systems' are going to be considered in this paper:

1. The retention criterion g_r (3) composed by two failure modes g_{r_a} and g_{r_b} ((5) and (6)) (group 1 soils).
2. Filter system where only the failure of the filter is considered. It is composed by the following failure modes: $M_F \in \{g_u, g_{a_i};\ i \in \{5, 15, 30, 50, 70, 85\}$ and $g_c\}$.
3. Global filter-base system where we consider all the practical criteria M_{F+B}.

The first system is a parallel system because it fails if **all** the components (criteria) fail simultaneously. The probability of failure of a parallel system is the intersection of the probabilities of failure for m modes:

$$P_{fp} = Pr[\cap_{k=1}^{M}\{X_k \leq -\beta_k\}] = \Phi_m(-\boldsymbol{\beta}, \boldsymbol{R}), \tag{16}$$

where $\boldsymbol{\beta}$ is the vector of reliability indexes for all the failure modes considered, $\boldsymbol{R}$ is the correlation matrix between any two failure modes and Φ_m is the multivariate normal distribution function.

The filter system is a series system because it fails if **any** of the components (criteria) fails. The probability of failure of a series system is the union of the probabilities of failure for m modes:

$$P_{fs} = Pr[\cup_{k=1}^{m}\{X_k \leq -\beta_k\}] = 1 - Pr[\cap_{k=1}^{m}\{-X_k \leq \beta_k\}] = 1 - \Phi_m(\boldsymbol{\beta}, \boldsymbol{R}). \tag{17}$$

The global filter-base system composed by all the practical criteria. For group1 soils is a series-parallel system fails if **any** of the following modes fails: $M_{F+B} \in \{g_r, g_p, g_u, g_{a_i};\ i \in \{5, 15, 30, 50, 70, 85\}$ and $g_c\}$, where g_r fails if all its components g_{r_a} and g_{r_b} fail simultaneously. In the case of group 2 soils, is a series system.

Using first order system reliability theory, each non-linear limit state function g_m is approximated by linear functions in the normal random space:

$$g_m^* = \boldsymbol{a}_m^T \boldsymbol{z} + \beta_m;\ m = 1, \dots, M, \tag{18}$$

where $\boldsymbol{z}$ is the vector of uncorrelated standard normal random variables, and $\boldsymbol{a}_m = (-z_{m1}/\beta_m, \dots, -z_{mn}/\beta_m)^T$ is the vector of normalized influence coefficients for the mth mode with $\boldsymbol{a}_m^T \boldsymbol{a}_m = 1$. The correlation matrix elements R_{ij} are evaluated using $R_{ij} = \boldsymbol{a}_i^T \boldsymbol{a}_j$.

Clearly, the computation of the multinormal integrals is a necessary step for estimating the probability of failure of structural systems. There are several approaches (see Melchers (1999)) but in this paper the product of conditional marginals (PCM) method (see Pandey (1998)) that shows high accuracy and simplicity of computation is used.

Whereas the evaluation of the probability of failure for systems using expressions (16) and (17) and first order reliability concepts is approachable with a sufficient accuracy for most practical applications, the evaluation of the probability of failure of a series-parallel system (for group 1 soils) is a very difficult task, and Monte Carlo simulation methods will be used.

2.4 Statistical Assumptions

To perform a probabilistic study in the filter-base system, the joint probability density function of all variables is required. As the size of the particles in the base and filter can not have negative values, the particle sizes are modeled as log-normal random variables.

Filter and base particle sizes are considered independent one form each other, this hypothesis is reasonable because filter and base materials usually come from different sources. However, as filter material comes from the same source, their particle sizes are not independent and the values of the correlation coefficients ($\boldsymbol{\rho}$) are required.

In this paper, for the sake of simplicity, no other sources of uncertainty are considered, but other random variables could be added easily for considering model uncertainty, for instance, variables F_m could be considered random. Obviously, this assumption would require the calibration using laboratory test results in order to establish an adequate distribution.

3 Analytical solution of the reliability problems

One advantage of using FORM ("First Order Reliability Methods") and considering that, (i) the limit state equations given in (5)-(10) are linear, and (ii) the statistical assumptions given in subsection 2.4, is that problem (11)-(13) can be solved analytically. The reliability problems related to the different failure modes can be expressed as follows:

$$\beta = \underset{x,y}{\text{Minimum}} \quad \sqrt{z_1^2 + z_2^2}\ , \tag{19}$$

subject to

$$z_1 = \frac{\ln x + \ln\sqrt{1+v_x^2}}{\sqrt{\ln(1+v_x^2)}}, \tag{20}$$

$$z_2 = \frac{\ln y - \ln\sqrt{1+v_y^2} - \rho z_1\sqrt{\ln(1+v_y^2)}}{\sqrt{\ln(1+v_y^2)}\sqrt{1-\rho^2}}, \tag{21}$$

$$g = a - \frac{1}{t}\frac{y}{x} = 0. \tag{22}$$

The optimal solution of problem (19)-(22) is:

$$x^* = \frac{1}{at}\exp\left(\frac{(\ln at - l_x)l_y + \rho\sqrt{l_x l_y}(0.5(l_x+l_y) - \ln(at))}{l_x - 2\rho\sqrt{l_x l_y} + l_y}\right), \tag{23}$$

$$y^* = \exp\left(\frac{(\ln at - l_x)l_y + \rho\sqrt{l_x l_y}(0.5(l_x+l_y) - \ln(at))}{l_x - 2\rho\sqrt{l_x l_y} + l_y}\right), \tag{24}$$

where $l_x = \ln(1+v_x^2)$ and $l_y = \ln(1+v_y^2)$. Using (19)-(21) the corresponding reliability index β^* is obtained. With regards to (12)-(13), the point $\boldsymbol{\eta}_1 = (x^{(1)}, y^{(1)})^T = (1/\sqrt{(1+v_x^2)},\, 1/\sqrt{(1+v_y^2)})^T$ is obtained substituting in (20)-(21), z_1 and z_2 by 0. Using the verification equation (22) particularized for $\boldsymbol{\eta}_1$, the sign of the reliability index will be obtained as follows:

$$g_1 = a - \frac{1}{t}\frac{y^{(1)}}{x^{(1)}} = a - \frac{1}{t}\frac{\sqrt{(1+v_x^2)}}{\sqrt{(1+v_y^2)}} \begin{cases} \text{if } g_1 > 0 & \text{then} \quad \beta^* = \beta^* \\ \text{if } g_1 \leq 0 & \text{then} \quad \beta^* = -\beta^*. \end{cases} \tag{25}$$

Note that when $g_1 > 0$, the point $\boldsymbol{z}_1$ is in the safe region and the sign obtained from solving (19)-(22) is right. Otherwise, the point $\boldsymbol{z}_1$ is in the failure region and the sign changes (probability of failure bigger than 0.5).

For example, for the filter uniformity criteria, and comparing to problem (19)-(22), it can be concluded that $x = D_{10F}/\mu_{D_{10F}}$, $y = D_{60F}/\mu_{D_{60F}}$, $t = \mu_{D_{10F}}/\mu_{D_{60F}}$, and $a = 20$. Substituting in (23)-(25) the solution the uniformity reliability problem is obtained.

The solution of problem (11)-(13) associated with verification equations that involve only one particle size distribution variable (r_b, c) is straightforward.

3.1 Retention failure

As the most important failure mode from the safety point of view is, usually, the retention criteria (r), which in the case of group 1 soils involve two verification equations (r_a and r_b), we will solve and analyze the solution for different values of the variables involved.

Considering first the retention criteria (r_a), and comparing to problem (19)-(22), it is shown that $x = d_{85B}/\mu_{d_{85B}}$, $y = D_{15F}/\mu_{D_{615F}}$, $t = \mu_{d_{85B}}/\mu_{D_{15F}}$, and $a = 9$. Substituting in (23)-(25) the solution the uniformity reliability problem is obtained.

The problem associated with the second retention constraint (r_b) is:

$$\beta_{r_b}(\boldsymbol{\theta}) = \underset{D_{15F}}{\text{Minimum}} \quad \sqrt{z_{2,r_b}^2}\ ,$$

subject to

$$\begin{aligned} z_{2,r_b} &= \frac{\ln D_{15F} - \mu_{\ln D_{15F}}}{\sigma_{\ln D_{15F}}} = \frac{\ln D_{15F} - \ln\left(\mu_{D_{15F}}/\sqrt{1+v_{D_{15F}}^2}\right)}{\sqrt{\ln(1+v_{D_{15F}}^2)}}, \\ g_{r_b} &= 0.2 - D_{15F} = 0, \end{aligned}$$

whose optimal solution is:

$$D_{15F}^* = 0.2; \quad z_{2,r_b}^* = \frac{\ln 0.2 - \ln\left(\mu_{D_{15F}}/\sqrt{1+v_{D_{15F}}^2}\right)}{\sqrt{\ln(1+v_{D_{15F}}^2)}}; \quad \beta_{r_b}^* = z_{2,r_b}^*. \tag{26}$$

Note that the sign of the reliability index β_{r_b} coincides with the z_{2,r_b} sign.

Once the reliability related to the failure modes r_a and r_b has been obtained, the parallel system reliability has to be calculated as it is shown in Section 2.3:

$$P_{f_r} = \Phi_m(-\boldsymbol{\beta}_r, \boldsymbol{R}_r), \tag{27}$$

where

$$\boldsymbol{a}_{r_a} = \left(-\frac{z^*_{1,r_a}}{\beta_{r_a}}, -\frac{z^*_{2,r_a}}{\beta_{r_a}}\right)^T, \tag{28}$$

$$\boldsymbol{a}_{r_b} = \left(-\frac{z^*_{1,r_b}}{\beta_{r_b}}, -\frac{z^*_{2,r_b}}{\beta_{r_b}}\right)^T = (0,-1)^T, \tag{29}$$

$$\boldsymbol{R}_r = \begin{pmatrix} 1 & z^*_{2,r_a}/\beta_{r_a} \\ z^*_{2,r_a}/\beta_{r_a} & 1 \end{pmatrix}, \tag{30}$$

$$\boldsymbol{\beta}_r = (\boldsymbol{\beta}_{r_a}, \boldsymbol{\beta}_{r_b})^T. \tag{31}$$

Expression (27) can be solved using the product of conditional marginals (PCM) method (see Pandey (1998)) as follows:

$$P_{f_r} = \Phi_m(-\boldsymbol{\beta}_r, \boldsymbol{R}_r) = \Phi(-\beta_{r_a})\Phi\left(\frac{-\beta_{r_b} - \mu_{r_b|r_a}}{\sigma_{r_b|r_a}}\right), \tag{32}$$

where $\mu_{r_b|r_a}$ and $\sigma_{r_b|r_a}$ are the conditional mean and standard deviation, respectively, given by:

$$\mu_{r_b|r_a} = -\frac{z^*_{2,r_a}}{\beta_{r_a}}\frac{\phi(-\beta_{r_a})}{\Phi(-\beta_{r_a})}, \tag{33}$$

$$\sigma_{r_b|r_a} = \sqrt{1 - \left(\frac{z^*_{2,r_a}}{\beta_{r_a}}\right)^2 \frac{\phi(-\beta_{r_a})}{\Phi(-\beta_{r_a})}\left(-\beta_{r_a} + \frac{\phi(-\beta_{r_a})}{\Phi(-\beta_{r_a})}\right)}. \tag{34}$$

Therefore, the system reliability index associated with the retention criterion is $\beta_r = -\Phi^{-1}(P_{f_r})$.

Figure 2 shows the contours plots of the retention criteria reliability index β_{r_a} for different values of the ratio $\mu_{D_{15F}}/\mu_{d_{85B}}$ corresponding to group 1 soils, and the coefficients of variation $v_{D_{15F}}$ and $v_{d_{85B}}$, respectively. Owing to the exponential nature of the solution (23)-(24), the absolute value of the reliability index β_{r_a} tends to infinity when the coefficients of variation $v_{D_{15F}}$ and $v_{d_{85B}}$ tend to cero except when $\mu_{D_{15F}}/\mu_{d_{85B}} = 9$, which tends to cero. The reliability indexes corresponding to $v_{D_{15F}} = 0.01$ and $v_{d_{85B}} = 0.01$ are shown in all the graphs. Note that the reliability index is positive when $\mu_{D_{15F}}/\mu_{d_{85B}} < 0$, which means that the probability of failure is lower than 50% because the point of maximum likelihood $\left(\mu_{d_{85B}}/\sqrt{(1+v^2_{d_{85B}})}, \mu_{D_{15F}}/\sqrt{(1+v^2_{D_{15F}})}\right)$ is inside the safe region, and negative when $\mu_{D_{15F}}/\mu_{d_{85B}} < 0$, which means that the probability of failure is greater than 50% (point of maximum likelihood inside the failure region)

4 Sensitivity analysis

The problem of sensitivity analysis in reliability based problems has been discussed by several authors, see, for example, Frangopol (1985); Enevoldsen (1994); Sorensen and Enevoldsen (1992); Mínguez (2003); Mínguez et al. (2005, 2004). In this section it is shown how duality methods can be applied to sensitivity analysis in a straightforward manner. The method to be presented in this section is of general validity. The basic idea is simple. Assume that the sensitivity of the objective function to changes in some data values is looked for. Converting the data into artificial variables and locking them, by means of constraints, to their actual values, a problem that is equivalent to the initial optimization problem but has a constraint such that the values of the dual variables associated with them give the desired sensitivities is obtained.

To this aim, in problem (11)-(15), variable $\boldsymbol{\theta}$ is replaced by the artificial variable $\boldsymbol{\theta}^*$ and the following constraint is added:

$$\boldsymbol{\theta}^* = \boldsymbol{\theta} : \ \boldsymbol{\lambda}_m. \tag{35}$$

Converting the data $\boldsymbol{\theta} = (\tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ into artificial variables $\boldsymbol{\theta}^*$ and setting them, by means of constraint (35), to their actual values $\boldsymbol{\theta}$, the values of the dual variables $\boldsymbol{\lambda}_m$, associated with constraint (35) multiplied by $\text{sign}(u_m)$ give the sensitivities of the reliability indexes for mode m with respect to $\boldsymbol{\theta} = (\tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$. These sensitivities allow determining for example how the reliability of an engineering design changes when its design values and the statistical parameters of the random variables involved are modified.

If the sensitivities ($\boldsymbol{\mu}_m$) of the probability of failure (using FORM) with respect the data parameters are looked for the following formula should be used:

$$\boldsymbol{\mu}_m = \text{sign}(u_m)\frac{\exp(-\beta_m^2/2)}{\sqrt{2\pi}}\boldsymbol{\lambda}_m. \tag{36}$$

Note that this method requires the use of optimization techniques for solving the reliability problems related to the different failure modes (11)-(13), alternative methods for sensitivity analysis that would allow the use of the analytical expressions given in Section 3 are shown in Castillo et al. (2006, 2005).

5 Practical Examples

In this Section, the proposed reliability assessment method is carried out on two different examples: case study A, which uses a group 1 base soil, and case study B, which uses the Balderhead Dam data (group 2 base soil).

5.1 Case Study A: (Group 1 Base Soil)

In this example, both the filter and the base soil grain size distributions have been obtained from a real dam but their values have been intentionally modified to best represent the performance of the method.

5.2 Statistical data

To perform the probabilistic study in the filter-base systems, the joint probability density function of all variables is required. Considering the statistical assumptions stated in Section 4, the values of the statistical parameters for the filter and the base are shown in Table 1, respectively. Note that it contains the mean value and the coefficient of variation of the different filter and base particle sizes involved in the reliability assessment. The grain curves are shown in Figure 3, where the mean filter and base gradations are shown with thicker black lines. The values of the correlation coefficients of the filter ($\boldsymbol{\rho}$) are shown in Table 2. Note that the natural logarithms of the different particle sizes involved in some of the practical design criteria show an important correlation. The correlations between particle sizes that are not directly related by the design criteria are not considered in the calculations.

Note that d_{85B} is less than 0.074 mm, therefore, retention criteria for soil group 1 must be used.

5.3 Reliability assessment results

First of all, reliability indexes and probabilities of failure are evaluated solving problem (11)-(15) for each failure mode. The solution could be obtained using both numerical methods or the analytical solution presented in Section 3. We have used numerical methods in order to obtain the sensitivity analysis at the same time.

The proposed method has been implemented in GAMS (General Algebraic Modelling System) (see Castillo et al. (2001)) using the generalized reduced gradient method (for more details, see Vanderplaats (1984) or Bazaraa et al. (1990)) that has shown good convergence properties when the variables are constrained.

The results of the proposed method are given in Table 3, where β_m is the reliability index for mode m, $\boldsymbol{\eta}$ is the design point, failure point or point of maximum likelihood in the original random space, which is the point whose density function value under the statistical assumptions made is the biggest. It represents the most likely values of the random values where failure occurs. $\boldsymbol{z}$ is the design point in the standard normal random space. Note that both are related by means of the Rosenblatt transformation (12), and P_m is the probability of failure for each practical criteria using either FORM methods, Monte Carlo simulation or both. With respect the Monte Carlo simulation it is worth mentioning that 10^6 sample points were used.

One advantage of the practical criteria is the simplicity of the limit state equations, which considering the random variables as in Section 2.4, allows making the graph of the bi-variate standard normal random variable in 2-D and 3-D. In Figure 4 the probability density function contours, the limit state equations r_a and r_b (linear), the design points and the reliability indexes β_{r_a} and β_{r_b} in the standard normal random space are shown. Note that the FORM method is exact for linear limit state equations.

Table 3 also shows the system reliability evaluations using the PCM method, which has been implemented in Matlab. The β_m-values and the correlation matrix elements R_{ij} are used. Note that the correlation elements are obtained using $R_{ij} = \boldsymbol{a}_i^T \boldsymbol{a}_j$, where $\boldsymbol{a}_i^T = (-z_{i1}/\beta_i, -z_{i2}/\beta_i)$. Note that the term 'correlation' used in the statement refers to correlation between failure modes, different from the correlation between filter particle sizes.

From Table 3 the following conclusions can be withdrawn:

1. The most important failure mode is the retention criteria with a system probability of failure of $\approx 22.07\%$.

2. The failure of the filter itself is less probable, but the probability of the percentage of fines passing 0.075 mm sieve being greater than 5% is significant ($\approx 7\%$). Additional test should be perform to verify if they are plastic.

3. The probability of failure associated with the filter uniformity criteria and autostability between diameter filters are negligible.

4. The global probability of non-fulfilling the practical criteria is $\approx 27.43\%$.

The method also gives the sensitivities associated with the β-values. Only the sensitivities of the retention criteria reliability index is shown in Table 4. The term $\frac{\partial \beta_{r_a}}{\partial x}$ represents the change in the reliability index β_{r_a} when the data x increases one unit, whereas the term $\frac{\partial \beta_{r_a}}{\partial x}|x|$ is the relative sensitivity, which allows comparing sensitivities between parameters with different magnitudes, the bigger the value the more sensitive the parameter is. It is useful to know how much the β-values change due to a small change in a single data value (e.g., the means or the coefficients of variation).

Note, for example, that a unit increase in the coefficients of variation $v_{D_{F15}}$ and $v_{d_{B15}}$ (uncertainty increase) leads to a -0.105 and -2.248 decrease of the retention reliability index (β_{r_a}) (see the corresponding entrance $\frac{\partial \beta_{r_a}}{\partial x}$ in Table 4), respectively.

As it is expected, for example, increasing the uncertainty in the the retention criterion r_a (coefficient of variation) decreases the reliability index increasing the probability of failure.

5.4 Case study B: Balderhead dam (Group 2 base soil)

In this Section, in order to add comparisons with experimental data form the literature, the proposed reliability assessment method is performed using data from a real dam. In this case Balderhead dam data (group 2 base soil) has been selected.

Balderhead Dam is located in Northern England. Designed in 1959 and completed in 1965, the clay core cracked by hydraulic fracture just before the reservoir reached top water level in February 1966. Internal erosion followed and, fourteen months later, two sink holes developed at the crest over the upstream boundary of the clay core. The failure and subsequent investigation of Balderhead Dam is described in detail by (Vaughan and Soares, 1982). A crack erosion model and the reduced Particle Size Distribution method were used by (Locke, 2001) to predict why the dam failed.

5.5 Statistical data

Considering the statistical assumptions stated in Section 4, the values of the statistical parameters for the filter and the base are shown in Table 5, respectively. Note that it contains the mean value and the coefficient of variation of the different filter and base particle sizes involved in the reliability assessment. This data are obtained considering that the logarithm of the maximum value corresponds to the 98% quantile. According to them, Balderhead Dam core soil has between 70% and 40 % passing 0.075 mm sieve, thus, soil group 2 retention criteria has to be used.

5.6 Reliability assessment results

The results of the proposed method are given in Table 6, from this table the following conclusions can be withdrawn:

1. The most important failure mode is the retention criteria with a probability of failure of $\approx 79.3\%$.

2. The failure of the filter itself is very high $\approx 80\%$.

3. The global probability of non-fulfilling the practical criteria is $\approx 99\%$.

Note that the method shows that the probability of failure for this particular case is very high, confirming the risk associated with its filter design.

6 Conclusions

The method presented in this paper is specially suitable for assessing the probability of fulfilling the empirical criteria considering the statistical behavior of the filter and base soil particle size, respectively. This method allows practical engineers establish the safety level of existing filters to know how far the filter is from fulfilling the established practical criteria. In addition, sensitivity analysis can be easily performed by transforming the input parameters into artificial variables, which are constrained to take their associated constant values.

Additional advantages of the proposed method include:

1. The method is simple and allows and easy connection with optimization frameworks.
2. Statistical variability of filter and base size is taken into account allowing to determine the safety level of the filter by means of the probability of nonfulfillment of individual criteria or the total system.
3. It is very flexible allowing easy substitution of the limit state equations associated with the different failure modes.
4. This method allows comparison between filters in different dams in order to prioritize the rehabilitation investments.
5. Sensitivity values with respect to the target reliability indexes (or probabilities of failure) are given, without additional effort, by the values of the dual variables. This allows to identify the most important variables to be controlled, which might be very useful during construction.
6. It allows obtaining sensitivity values with respect the data samples. This provides a useful tool for outlier or erroneous data detection.

Finally, the methodology it is expected to be a very useful tool in order to approach probabilities of failure in the context of risk assessment of embankment dams. With that purpose, future research should be focused on estimating load (hydraulic gradients) probabilities and transforming the existing empirical criteria in real limit state equations.

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Table 1: Filter D_{xF} and Base d_{xB} values (mm) for the practical example (group 1 soil).

ref.	D_{5F}	D_{10F}	D_{15F}	D_{20F}	D_{30F}	D_{45F}	D_{50F}	D_{60F}	D_{65F}	D_{70F}	D_{85F}	D_{100F}	d_{15B}	d_{85B}
μ_X	0.093	0.163	0.210	0.267	0.395	0.690	0.890	1.419	1.709	2.056	3.319	10.000	0.001	0.0314
v_X	0.136	0.093	0.074	0.088	0.075	0.124	0.143	0.097	0.084	0.076	0.035	0.000	0.497	0.392
$\mu_{\ln X}$	-2.388	-1.818	-1.565	-1.325	-0.932	-0.379	-0.127	0.345	0.532	0.718	1.199	2.303	-7.001	-3.531
$\sigma_{\ln X}$	0.136	0.093	0.074	0.088	0.075	0.123	0.142	0.097	0.084	0.076	0.035	0.000	0.470	0.378

Table 2: Correlations between the involved filter particle sizes D_{xF} (group 1 soil).

	D_{5F}	D_{10F}	D_{15F}	D_{20F}	D_{30F}	D_{45F}	D_{50F}	D_{60F}	D_{65F}	D_{70F}	D_{85F}	D_{100F}
D_{5F}	1.000	–	–	0.337	–	–	–	–	–	–	–	–
D_{10F}	–	1.000	–	–	–	–	–	0.145	–	–	–	–
D_{15F}	–	–	1.000	–	0.869	0.521	–	–	–	–	–	–
D_{20F}	0.337	–	–	1.000	–	–	–	–	–	–	–	–
D_{30F}	–	–	0.869	–	1.000	0.865	–	–	–	–	–	–
D_{45F}	–	–	0.521	–	0.865	1.000	–	–	–	–	–	–
D_{50F}	–	–	–	–	–	–	1.000	–	0.968	–	–	–
D_{60F}	–	0.145	–	–	–	–	–	1.000	–	–	–	–
D_{65F}	–	–	–	–	–	–	0.968	–	1.000	–	–	–
D_{70F}	–	–	–	–	–	–	–	–	–	1.000	0.965	–
D_{85F}	–	–	–	–	–	–	–	–	–	0.965	1.000	–
D_{100F}	–	–	–	–	–	–	–	–	–	–	–	1.000

Table 3: Independent failure criteria reliability assessment for the practical example (group 1 soil).

m	β_m	z_1	z_2	$\boldsymbol{\eta}$		P_m (FORM)	P_m (Monte Carlo)
r_a	0.598	-0.586	0.115	0.023	0.211	0.27506	0.27512
r_b	-0.603	–	-0.603	–	0.200	0.72690	0.72686
p	8.518	8.414	-1.331	0.047	0.189	0.00000	0.00000
u	6.706	-4.239	5.197	0.110	2.193	0.00000	0.00000
a_5	4.059	-3.206	2.489	0.059	0.297	0.00002	0.00002
a_{15}	25.547	-5.928	24.849	0.135	0.673	0.00000	0.00000
a_{30}	15.206	6.887	13.557	0.661	3.307	0.00000	0.00000
a_{50}	14.637	-13.835	4.776	0.123	0.614	0.00000	0.00000
a_{70}	26.105	-25.515	5.520	0.294	1.472	0.00000	0.00000
a_{85}	14.440	-14.440	0.000	2.000	10.000	0.00000	0.00000
c	1.486	-1.486	–	0.075	–	0.06865	0.06899
System reliability							
r	–	–	–	–	–	0.22068	0.22063
M_F	–	–	–	–	–	0.06865	0.06901
M_{F+B}	–	–	–	–	–	–	0.27429

Table 4: Reliability index sensitivity with respect the parameters for the retention criteria (group 1 soil).

x	$\mu_{d_{B85}}$	$\mu_{D_{F15}}$	$v_{d_{B85}}$	$v_{D_{F15}}$	$--$	F_{r_a}
$\frac{\partial \beta_{r_a}}{\partial x}$	82.478	-12.362	-2.248	-0.105	$--$	0.288
$\frac{\partial \beta_{r_a}}{\partial x}\|x\|$	2.593	-2.593	-0.882	-0.008	$--$	2.593

Table 5: Filter D_{xF} values (mm) for the Balderhead dam filter (group 2 soil).

Filter ref.	D_{5F}	D_{10F}	D_{15F}	D_{20F}	D_{30F}	D_{45F}	D_{50F}	D_{60F}	D_{65F}	D_{70F}	D_{85F}	D_{100F}
μ_X	0.82	1.19	2.01	2.72	5.13	8.55	9.84	13.55	15.59	20.29	35.62	50.68
v_X	0.92	1.10	1.03	0.92	0.80	0.70	0.72	0.52	0.49	0.37	0.19	0.16
$\mu_{\ln X}$	-0.51	-0.22	0.34	0.69	1.39	1.95	2.08	2.48	2.64	2.94	3.56	3.91
$\sigma_{\ln X}$	0.78	0.89	0.85	0.78	0.70	0.63	0.64	0.49	0.46	0.36	0.19	0.16

Table 6: Independent failure criteria reliability assessment for the Balderhead dam example (group 2 soil).

m	β_m	z_1	z_2	$\boldsymbol{\eta}$		P_m (FORM)	P_m (Monte Carlo)
r_b	-0.817	–	-0.817	–	0.700	0.79296	0.79317
u	0.282	-0.247	0.136	0.642	12.834	0.38887	0.38896
a_5	0.366	-0.259	0.259	0.490	2.449	0.35724	0.35782
a_{15}	0.507	-0.390	0.324	1.005	5.026	0.30595	0.30579
a_{30}	1.108	-0.824	0.741	2.238	11.189	0.13385	0.13331
a_{50}	1.323	-1.073	0.774	4.011	20.055	0.09294	0.09276
a_{70}	2.448	-2.176	1.122	8.634	43.169	0.00718	0.00720
a_{85}	5.040	-3.790	3.322	17.233	86.164	0.00000	0.00000
c	2.654	-2.654	–	0.075	–	0.00398	0.00398
				System reliability			
M_F	–	–	–	–	–	0.797653	0.798369
M_{F+B}	–	–	–	–	–	0.989798	0.989922

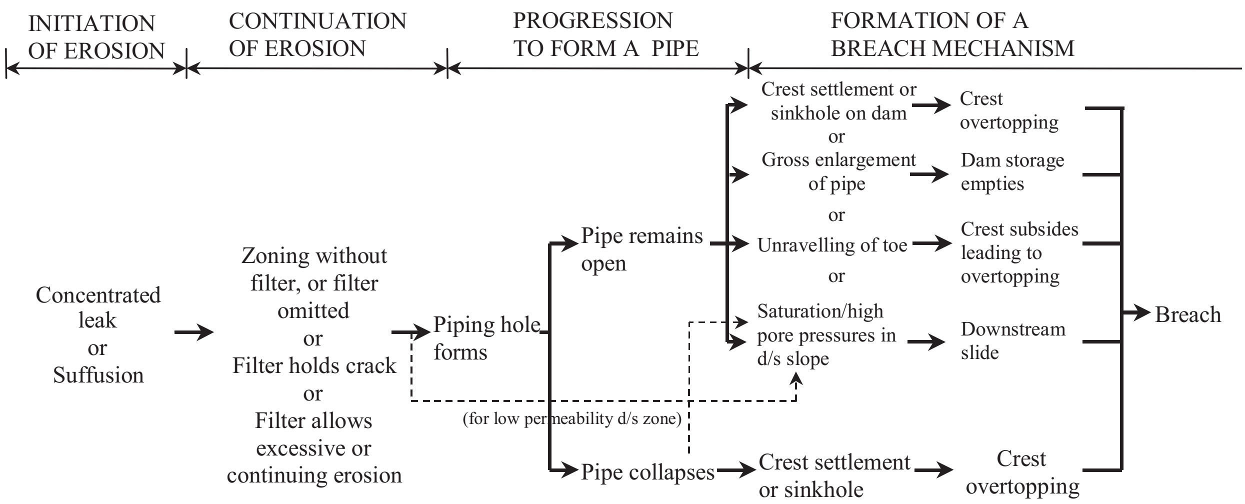

Figure 1: Failure path diagram for failure by pipping through the embankment (see Foster and Fell (1999)).

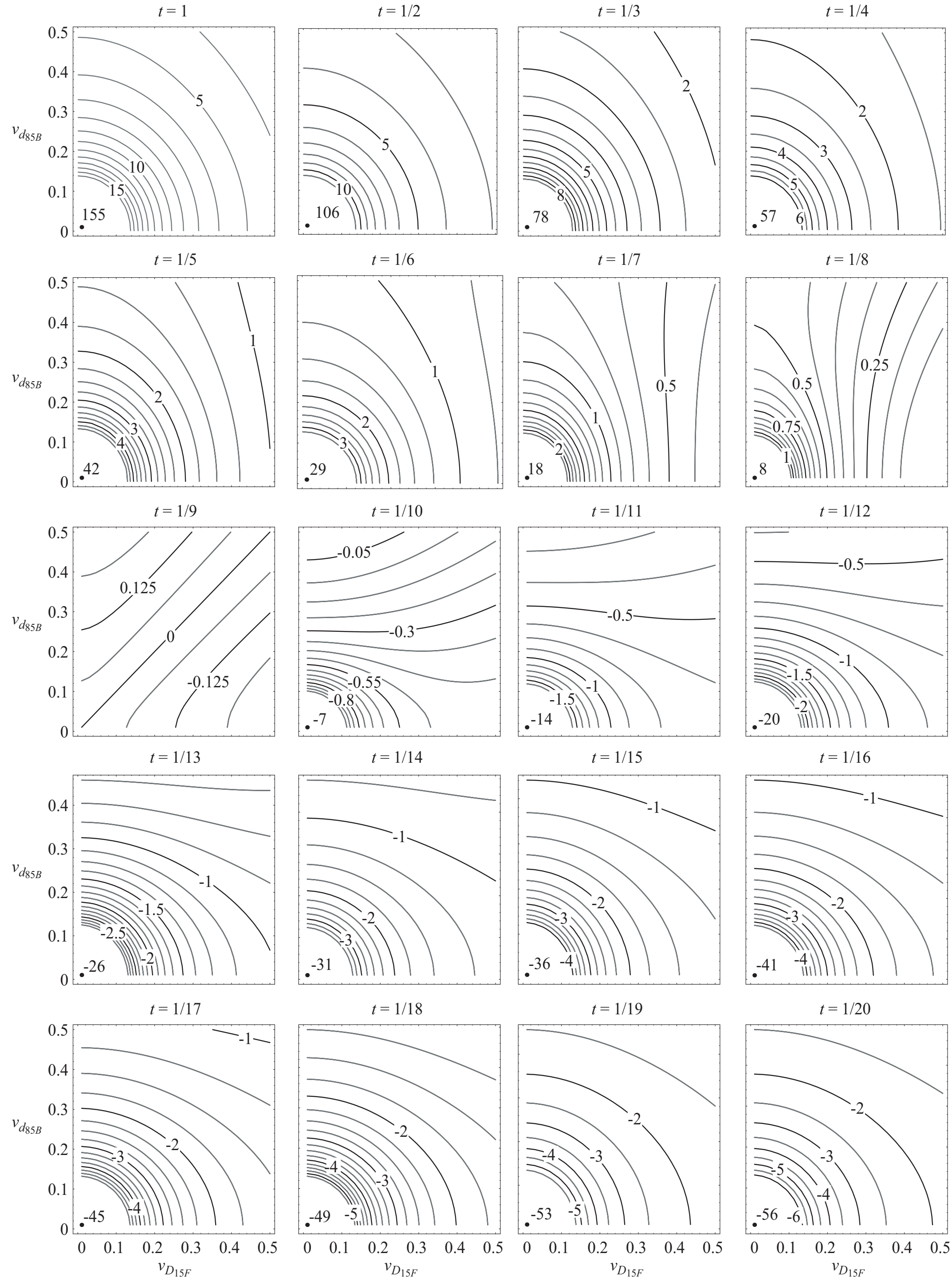

Figure 2: Contours plots of the retention criteria reliability index β_{r_a} for different values of the ratio $t = \mu_{d_{85B}}/\mu_{D_{15F}}$, and $v_{D_{15F}}$ and $v_{d_{85B}}$, respectively.

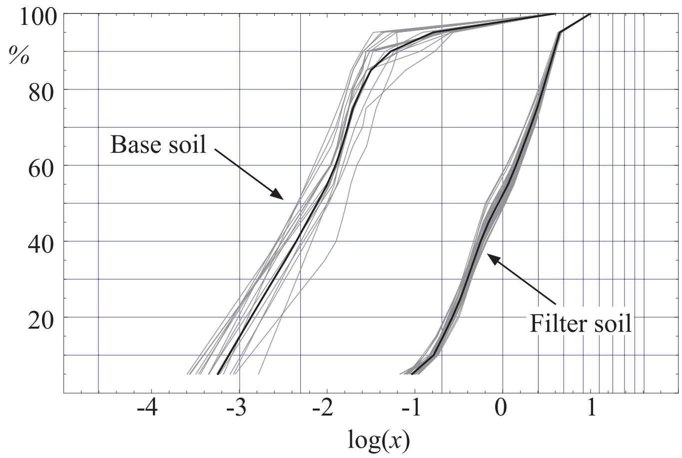

Figure 3: Sample grain curves of the filter and base particle sizes (gray lines) for the practical example (group 1 soil).

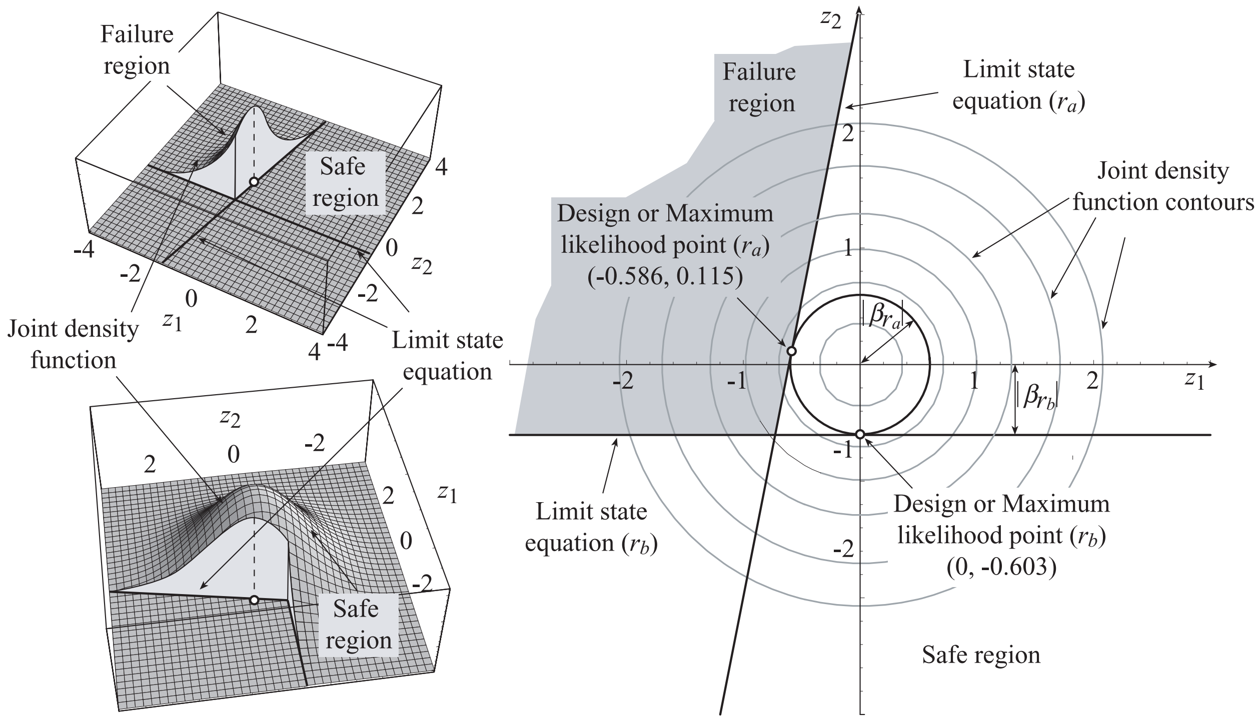

Figure 4: 2-D and 3-D graphical illustration of the reliability problem associated with the retention criteria r for the practical example (group 1 soil).