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Exploring the interannual variability of extreme wave climate in the Northeast Atlantic Ocean.

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Abstract

The extreme wave climate is of paramount importance for: i) off-shore and coastal engineering design, ii) ~~ship design and~~ maritime transportation, ~~or~~ ii) analysis of coastal processes. Identifying the synoptic patterns that produce extreme waves is necessary to understand the wave climate for a specific location. Thus, a characterization of these weather patterns may allow the study of the relationships between the magnitude and occurrence of extreme wave events and the climate system.

The aim of this paper is to analyze the interannual variability of extreme wave heights. For this purpose, we present a methodological framework and its application to an area over the North East (NE) Atlantic Ocean. The climatology in the NE Atlantic is analyzed using the self-organizing maps (SOMs). The application of this clustering technique to monthly mean sea level pressure fields provides continuum of synoptic categorizations compared with discrete realizations produced through most traditional methods.

The extreme wave climate has been analyzed by means of monthly maxima of the significant wave height (SWH) in several locations over the NE Atlantic. A statistical approach based on a time-dependent generalized extreme value (GEV) distribution has been applied. The seasonal variation was characterized and, afterwards, the interannual variability was studied throughout regional pressure patterns. The anomalies of the 50-year return level estimates of SWH, due to interannual variability have been projected into the weather types of SOM. It provides a comprehensive visual representation, which relates the weather type with the positive or negative contribution to extreme waves over the selected locations.

Keywords

weather types, generalized extreme value distribution, climate variability, synoptic classification, Self Organizing Maps, extreme wave climate

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Eliminado: A simple graphical representation explaining the interannual variability of extreme waves due to different synoptic patterns is then provided.

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76 1. INTRODUCTION

77 The most severe conditions of wave climate are of paramount importance on
78 natural coastal processes (i.e. sediment transport or the development of the seaweed
79 meadows), coastal management and engineering design (maritime works, ship design,
80 route definition, offshore structures design, operability,...). Thus, there is a need for
81 appropriate methods to describe these phenomena.

82 During the last decades, the study of the extreme wave climate has increased
83 significantly. The statistical modelling of the extreme wave height including seasonal
84 and interannual variability have been studied by numerous authors (Wang et al. 2001,
85 Caires et al. 2006, Méndez et al. 2006, Menéndez et al. 2009, Izaguirre et al. 2010,
86 Hemer, 2010). However, there is not a clear conclusion about the atmospheric situations
87 that cause the interannual fluctuations on extreme wave heights. From this point of
88 view, the aim of this work is to analyse the variability in the state of the atmosphere,
89 and to investigate if these variations can explain or help to understand the complex
90 relationships between wave forcing at a regional scale, and their effect in the interannual
91 variability of the extreme wave climate at a local spatial scale.

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92 In the earliest 70's synoptic climatology was established as a climatological
93 subfield with the publication of 'Synoptic climatology: methods and applications'
94 (Barry and Perry, 1973). After that seminal work, a lot of techniques have been applied
95 to explore and analyze the climatology in order to understand and simplify data of
96 geophysical variables. Several statistical methods have been developed to relate
97 synoptic-scale atmospheric circulation to local environmental responses (analysing
98 variables like temperature, precipitation or pressure fields). The main advantage of the
99 statistical techniques is that a large amount of complex data fields (with spatial and

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100 temporal dimensions) can be processed automatically to output a simple and readable
101 synthesis, minimizing the human factors.

102 The principal component analysis (PCA) is one of the most popular techniques.
103 PCA is especially useful to reduce the number of dimensions and identify patterns in
104 environmental data. The data sample is projected in a space with minor dimension
105 where the vectors of the new orthogonal base maximize the variance of the data sample.
106 This technique removes the data dependency and data redundancy with the minimum
107 lost of variance, which is sometimes required by the assumptions of many statistical
108 methods.

109 The clustering methods try to reduce the amount of data by categorizing or
110 grouping similar data together. These methods are used to partition the sample data into
111 clusters defined by centroids or reference vectors representing the data in a more
112 compact and manageable way. The self-organizing maps (SOMs) is one of the most
113 powerful data mining techniques for clustering high-dimensional data due to its
114 graphical visualization properties. The cluster centroids are forced with a neighborhood
115 mechanism to a space with smaller dimension (usually a two-dimensional lattice)
116 preserving the topology of data in the original space. Therefore, the clusters are spatially
117 organized in the lattice of projection which gives an intuitive analysis of the information
118 contained in the data.

119 Several applications of these techniques can be found in the wave climate field
120 trying to explain relations of sea states with atmospheric patterns. Bacon and Carter
121 (1993) showed the relationship between wave heights and the north-south atmospheric
122 pressure in the North Atlantic (the so-called North Atlantic Oscillation, NAO). Later on,
123 Kushnir et al. (1997) found a link between the wintertime monthly significant wave
124 height (SWH) and monthly average sea level pressure (SLP) using a canonical

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125 correlation analysis. Wang and Swail (2001, 2002) applied a PCA on both the SLP and
126 extreme wave height anomalies in the Northern Hemisphere to analyse their correlation.
127 Woolf et al. (2002) shows that a large fraction of the wave height anomalies in the
128 northeastern sector of the Atlantic is associated to a single pattern of pressure anomalies
129 that resembles the NAO. Izaguirre et al. (2010) introduces the interannual variability in
130 the generalized extreme value (GEV) distribution of extreme wave climate via the
131 location parameter as linear covariates using principal components (PCs) of monthly sea
132 level pressure anomalies. Le Cozannet et al. (2011) analysed the influence of
133 teleconnection patterns in the interannual variability of the frequency of sea state modes
134 in the Bay of Biscay, obtained from a K-means classification.

135 Following the hypothesis that interannual variability of the extreme wave height is
136 induced by patterns in the atmospheric circulation, the aim of this work is to present a
137 methodological framework to explain the relationship between extreme wave height
138 anomalies and the synoptic situation that produces it by means of a graphical
139 representation. To achieve this goal, a SOM analysis is carried out to process the
140 principal components (PCs) of SLP of the NE Atlantic area, to characterize the
141 climatology on a bidimensional lattice. The extreme wave height statistics at six
142 different locations over the studied domain is modelled by applying a time-dependent
143 GEV model including seasonal and interannual variability. The topology preservation
144 property of the SOM allows defining a function on the SOM lattice corresponding to
145 average value of extreme wave height for the reanalysis SLP dates corresponding to
146 each of the clusters. The interannual variability of the extreme wave climate at each
147 location projected into the climatological lattice is used to study the relationship with
148 the synoptic states and to analyse how extreme wave probability distributions change
149 due to changes in climatic conditions.

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150 The paper is organized as follows. Section 2 provides a description of the SLP and
151 the wave data used. In Section 3 we present the methodology, describing the data
152 mining techniques, PCA and SOM, and the statistical modelling of the extreme wave
153 height. Results are shown in Section 4, the NE Atlantic weather types [issued from the](#)
154 [SOM analysis](#), extreme wave climate variability and the relationship between both are
155 presented. Finally, some conclusions are given in Section 5.

156

157 **2. DATA**

158 **2.1 Sea Level Pressure data**

159 The sea level pressure fields used in this work come from the reanalysis dataset of
160 the National Center for Environmental Prediction-National Center for Atmospheric
161 Research (NCEP-NCAR; Kalnay et al. 1996). The SLP data consist of 6-hourly fields
162 on a Gaussian grid [with T62 resolution \(about 210 km, for more details see Kalnay et al.](#)
163 [1996\)](#). The period of the reanalysis used in this study spans from 1948 to 2008.

164 The spatial domain under study spans from 25° N to 70° N and 60° W to 10° E
165 (see figure 1) using a 5° x 5° spatial resolution grid [where the SLP data are interpolated](#).
166 The area is selected to capture the action center of the NAO, which is the most
167 prominent oscillation mode in the North Atlantic. Monthly mean sea level pressure
168 (MSLP) is extracted for the regrided spatial domain. In summary, the monthly MSLP
169 data consist of a record of 744 monthly values from 1948 to 2008, each defined at 150
170 grid points.

171

172 **2.2 Wave data**

173 The wave data used in this work come from the global wave reanalysis database
174 GOW (Reguero et al. 2012). GOW reanalysis has been generated with the third

175 generation model WaveWatch III (Tolman 2010). The wave spectrum is computed by
176 integration of the energy balance equation without any prior restriction about the wave
177 spectral shape. The model is forced by 6-hourly wind fields from the atmospheric
178 reanalysis NCEP/NCAR (with T62 Gaussian grid resolution).

179 This database spans from 1948 onwards with hourly resolution, and $1.5^\circ \times 1^\circ$
180 (longitude x latitude) spatial resolution. A validation and calibration procedure was
181 applied by using instrumental measurements from both satellite and buoy records (more
182 details in Reguero et al. 2012, Mínguez et al. 2011, Mínguez et al. 2012).

183 Six locations in the east part of the North Atlantic basin are selected (see figure
184 1): i) a northern point (NP, lon= 15°W , lat= 55°N) around 150 km westward of Ireland,
185 ii) a point located close to the Bretagne coast in France (BR, lon= 7.5°W , lat= 49°N), iii)
186 a point in front of the Landes region, in the Gulf of Biscay (LA, lon= 1.5°W , lat= 44°N),
187 iv) a point in the northwest coast of Spain, in front of Coruña (CO, lon= 10.5°W ,
188 lat= 43°N), v) a location in front of Lisbon (LI, lon= 10.5°W , lat= 38°N), and finally, vi) a
189 point in the Azores Islands (AZ, lon= 27°W , lat= 39°N). The point of Landes is located in
190 intermediate waters (up to 100 m) while the rest of them are in deep water.

191

192 **3. METHODS**

193 **3.1 Summary of the approach**

194 In order to establish the relationship between extreme wave height anomalies and
195 the atmospheric forcing, we follow the next methodology:

196 1. Before applying any statistical technique we process the atmospheric data
197 standardizing the MSLP fields.

198 2. We apply Principal Component Analysis to the standardized SLP in order to
199 reduce dimensionality and identify dominant patterns of variability.

200 3. The atmospheric PCs are used for both modeling the interannual variability of
201 extreme wave height and clustering atmospheric patterns into weather types using SOM
202 technique.

203 4. For six selected locations we model the interannual variability of extreme wave
204 height using a time-dependent generalized extreme value model. We introduce the PCs
205 of MSLP as covariates to model interannual variability.

206 5. In line with the extreme wave height modeling we cluster the atmospheric PCs
207 using the SOM technique obtaining a lattice of representative weather types of the
208 North Atlantic.

209 6. Using the probability of occurrence of the SOM lattice we project the
210 interannual variability of extreme wave height and relate weather types (atmospheric
211 forcing) with extreme wave height anomalies.

212 The statistical techniques and extreme value model used are described next.

213

214 **3.2 Principal Component Analysis**

215 The Principal Component Analysis (see Preisendorfer and Mobley, 1988) is
216 carried out on the MSLP in order to reduce the dimensionality of the problem,
217 preserving the maximum of the sample variance. It is a classical statistical linear
218 compression method which gives an optimal (in a statistical sense) linear reduction of
219 dimension (Gutierrez et al. 2004). This statistical technique is widely used in
220 climatology to identify dominant patterns of variability and/or reduce dimensionality of
221 climate data (Smith et al. 1996).

222 The reduction of dimensionality is achieved by creating a new set of orthogonal
223 (hence uncorrelated) and ordered variables, the principal components, spanning the
224 maximum variance of the data (Jolliffe 2002). Let $X(t) = [X_1(t), X_2(t), \dots, X_p(t)]$ be an

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225 $n \times p$ data matrix, $\{X_i(t); i = 1, \dots, p; t = 1, \dots, n\}$ is a vector containing n (monthly) values
226 of the i^{th} centered predictor (to avoid problems due to different scales, the variable
227 monthly MSLP is previously standardized, related to the average over $n = 744$ instants,
228 for each grid point, obtaining monthly MSLP anomalies), and p is the number of
229 predictors (i.e., $p = 150$ grid points over covering the region 25°N-70°N, 60°W-10°E in
230 the NA area). PCs components are obtained by

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$$232 \quad Z_i(t) = \sum_{k=1}^p e_{ki} X_k(t), \quad i = 1, \dots, p; t = 1, \dots, n$$

(1)

234 where e_m are the elements (loadings) of the m^{th} eigenvector of the covariance matrix

$$236 \quad S = \frac{1}{n-1} X^T X$$

(2)

238 The analysis of the anomalies of monthly MSLP yields the spatial modes and their
239 temporal amplitudes. The first 10 modes, explaining more than 90 % of the variability,
240 are chosen. Note that the first two modes are correlated with the two prominent
241 teleconnection indices of the North Atlantic: North Atlantic Oscillation (NAO) and East
242 Atlantic (EA) pattern. The correlation between the first and second modes and the NAO
243 Index is $r_1^{NAO} = 0.704$ and $r_2^{NAO} = 0.381$, respectively. Regarding the EA, only the
244 correlation with the second mode is statistically significant and equal to $r_2^{EA} = 0.628$.

Eliminado: To avoid problems due to different scales, the variable monthly MSLP is previously standardized for each grid point before applying PCA, obtaining monthly MSLP anomalies.¶

246 3.3 Time-dependent extreme model

247 Latest advances in extreme value theory (see Coles 2001) allow a better
248 description of the natural climate variability of extreme events of geophysical variables,
249 specifically extreme wave height. In this work a time-dependent GEV model for
250 monthly maxima SWH including seasonal and interannual variability is used. We have
251 considered time-dependent location $\mu(t)$, scale $\psi(t) > 0$ and shape $\xi(t)$ parameters of
252 the GEV (Coles 2001), with cumulative distribution function (CDF) of H_t (monthly
253 maxima of the significant wave heights observed in month t) given by

254

$$255 F_t(H) = \begin{cases} \exp \left\{ - \left[1 + \xi(t) \left(\frac{H - \mu(t)}{\psi(t)} \right) \right]_+^{-1/\xi(t)} \right\} & \xi(t) \neq 0 \\ \exp \left\{ - \exp \left[- \left(\frac{H - \mu(t)}{\psi(t)} \right) \right] \right\} & \xi(t) = 0 \end{cases}$$

256

(3)

257 where $[a]_+ = \max[a, 0]$. The GEV distribution includes the three classical distribution
258 families of extreme value theory: Gumbel family ($\xi = 0$); Fréchet distribution ($\xi > 0$),
259 and Weibull family ($\xi < 0$).

260 Figure 2 shows the total population of SWH for each location and the monthly
261 maxima sample. A clear seasonal variation is observed in all the points (stronger in
262 north latitudes, North Point and Bretagne) and also a clear interannual variability can be
263 appreciated, with severe and mild years, due to the natural climate variability. Since
264 most of the variability is explained by seasonal behavior (Izaguirre et al. 2011) the
265 introduction of harmonic functions to model seasonality is used (Menéndez et al.,
266 2009). We let the model introduce the best number of harmonics in the three
267 parameters.

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268 On the other hand, the hypothesis that extreme wave climate is affected by
269 regional SLP patterns is used. We introduce the PCs of monthly MSLP in the NE
270 Atlantic obtained in section 3.2 as covariates to model interannual variability (Izaguirre
271 et al., 2010). We let the model introduce up to ten PCs as linear terms in the location
272 and scale parameter (we standardize the PCs to give all of them the same relative weight
273 in the extreme value model).

274 Mathematically, the model can be expressed as:

$$275 \mu(t) = \beta_0 + \sum_{i=1}^{P_\mu} [\beta_{2i-1} \cos(i\omega t) + \beta_{2i} \sin(i\omega t)] + \sum_{j=1}^{10} \beta_{PCj} Z_j(t)$$

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$$277 \tag{4}$$

$$278 \log[\psi(t)] = \alpha_0 + \sum_{i=1}^{P_\psi} [\alpha_{2i-1} \cos(i\omega t) + \alpha_{2i} \sin(i\omega t)] + \sum_{j=1}^{10} \alpha_{PCj} Z_j(t)$$

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$$279 \tag{5}$$

$$280 \xi(t) = \gamma_0 + \sum_{i=1}^{P_\xi} [\gamma_{2i-1} \cos(i\omega t) + \gamma_{2i} \sin(i\omega t)]$$

$$281 \tag{6}$$

282 where β_0 , α_0 and γ_0 are mean values; β_i , α_i and γ_i ($i > 0$) are the amplitudes of the
283 harmonics; $\omega = 2\pi$ year⁻¹; P_μ , P_ψ and P_ξ determine the number of sinusoidal
284 harmonics in a year; and t is given in years. The parameter β_{PCj} and α_{PCj} represents
285 the influence on the location and scale parameters per unit of standardized Z_j in a
286 particular month, t . The model selection is carried out using the pseudo-optimal method
287 explained in Minguez et al. (2010).

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288 The instantaneous quantile $H_q(t)$ associated with the return period $1/q$ can be
289 obtained using:

$$291 H_q(\mu(t), \psi(t), \xi(t)) = \begin{cases} \mu(t) - \frac{\psi(t)}{\xi(t)} \left[1 - \{-\log(1-q)\}^{-\xi(t)} \right] & \xi(t) \neq 0 \\ \mu(t) - \psi(t) \log\{-\log(1-q)\} & \xi(t) = 0 \end{cases}$$

292 (7)
293 where probability q is given by $F_t(H) = 1 - q$. Since seasonal and interannual
294 variability have been modeled, the quantile varies depending on the time within the year
295 and the year itself.

296 The interannual variation in the time-dependent quantile can be expressed as the
297 difference between the time-dependent quantile (H_q) and the seasonal-dependent
298 quantile (H_{qs}), where the seasonal-dependent quantile is calculated from a regression
299 model where only the seasonal variation is considered.

$$301 \delta H_q = H_q - H_{qs}$$

302 (8)
303 where δH_q is the time-dependent quantile anomaly.

305 3.4 Self-Organizing Maps

306 Interannual wave climate variability is dependent on large-scale dynamic in the
307 atmosphere-ocean system. In this study we are interested in whether there is a direct
308 relationship between synoptic climatology and extreme wave climate. SOM is,
309 therefore, the selected technique to establish synoptic patterns (weather types). It is a

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310 statistical method developed in the field of data mining to deal with huge amounts of
311 data efficiently. This analysis tool, from the field of artificial neural networks, supports
312 analysis of variability in large, multivariate and/or multidimensional data sets through
313 the creation of a spatially organized set of generalized patterns of variability from the
314 data. A SOM summarizes the high-dimensional data space in terms of a set of reference
315 vectors (cluster centers) having spatial organization corresponding to a two-dimensional
316 lattice. Note that we use the PC vectors Z_i instead the original data x_i to train the SOM
317 (Gutierrez et al. 2005) in order to eliminate noise from the signal.

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318 The SOMs analysis provides a complementary nonlinear alternative to more
319 frequently used but linear methods, such as PCA. SOM has several advantages,
320 including: i) it handles nonlinear relationships, and ii) it provides a robust interpolation
321 method in areas of the input space not present in the available training input. Another
322 benefit, when applied to atmospheric data, is that it supports the development of
323 synoptic climatologies with an arbitrary number of smoothly transitioning climate
324 states, in contrast to traditional synoptic classification techniques. The projection of the
325 results in a lattice with spatial organization makes it different to other technique, being a
326 more powerful tool due to the easy interpretation of the results by visual inspection.

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327 A SOM is formed by an arbitrary number of clusters (or centroids) C_k , where
328 $k = 1 \dots m$, (m is the number of clusters) located on a two-dimensional matrix for
329 visualization purposes, that are representative of the probability density function of the
330 input data. Each cluster C_k is associated with two vectors . First, the vector $c_k = (i_k, j_k)$
331 describes the position of cluster C_k on the matrix. Besides, each of the clusters C_k is
332 associated with a reference vector $v_k = (v_{k1}, \dots, v_{kn})$ in the space of data, where n is the
333 number of month, previously defined in section 3.2. The number of selected clusters

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334 dictates how much intra cluster spread is represented by the classes. A broader range of
335 patterns with more gradual differences is easily produced by increasing the number of
336 clusters.

337 A clear advantage of SOM is the way the set of reference vectors, best
338 representing different clusters within the data, is obtained. It uses an unsupervised
339 learning process which minimizes an overall within-cluster distance from the data
340 vectors, or patterns x_i , to the corresponding reference vectors

341

$$342 \quad \sum_{k=1 \dots m} \sum_{x_i \in C_k} \|x_i - v_k\|^2$$

343

(9)

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344 where N is the number of available patterns (744 monthly patterns for the period 1948-

345 2008). The aim of the training algorithm is iteratively adapting the reference vectors

346 minimizing (9). First, the SOM clusters are initialized to random values. Then, the batch

347 training proceeds in cycles: on each training cycle, a data sample x_i is considered and

348 the best matching reference vector v_k is obtained as the one minimizing the Euclidean

349 distance to the data vector:

350

$$351 \quad \|v_{w(i)} - x_i\| = \min_k \{\|v_k - x_i\|, k = 1, \dots, m\}$$

352

(10)

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353 Then, the reference vector of the winning cluster is moved towards the sample

354 vector based on a learning rate parameter in the algorithm. The learning rate controls

355 how fast this process occurs. a small value leads to a slow and smooth learning process,

356 while a high value produces a fast but unstable learning process (Gutierrez et al. 2005).

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357 This training process includes a neighborhood adaptation mechanism so that
 358 neighboring clusters of the winning reference vector in the 2D matrix space are also
 359 adapted towards the sample vector. The number of adjacent clusters that are modified is
 360 specified by the radius of the training area, and the amount of adjustment varies: i) in
 361 inverse proportion to the distance from the initially identified cluster, and ii) in
 362 proportion to the learning rate parameter.

363 As a consequence of the neighborhood algorithm, during the iterative training the
 364 SOM behaves like a flexible lattice folding onto the cloud formed by the data in the
 365 original n dimensional space. Both the learning rate and the neighborhood algorithm
 366 radius decrease monotonically with time, softening the folding process (a linear decay
 367 to zero is usually chosen for these functions). For a detailed description of the process,
 368 the reader is referred to Oja and Kaski 1999.

370 4. RESULTS

371 First we have computed the extreme wave climate analysis in each location of the
 372 NE Atlantic. Figure 3 shows, for the six locations of interest, the seasonal and
 373 interannual modeling of the extreme wave height. Left panels show seasonality results.
 374 Note the variation throughout the year of the seasonal-dependant location and scale
 375 parameters and the seasonal-dependent quantile associated with the 50-year return
 376 period. The annual cycle is clear in all locations, particularly in Bretagne and Lisbon.
 377 The North Point shows a slightly asymmetric annual cycle, with higher events in
 378 autumn (October-November), which is accounted for throughout the shape parameter.
 379 Landes shows a long severe season that spans from October-November to March, but it
 380 presents milder extreme wave climate than North Point, Bretagne and Coruña, which is
 381 at similar latitude ($H_{50} \approx 10$ m in winter). Coruña, Lisbon and Azores present similar

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Eliminado: Note that we use the PC vectors u_i instead the original data X_i to train the SOM (Gutierrez et al. 2005).

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3.3 Time-dependent extreme model¶

Latest advances in extreme value theory (see Coles 2001) allow a better description of the natural climate variability of extreme events of geophysical variables, specifically extreme wave height. In this work a time-dependent GEV model for monthly maxima SWH including seasonal and interannual variability is used. We have considered time-dependent location $\mu(t)$, scale $\psi(t) > 0$ and shape $\xi(t)$ parameters of the GEV (Coles 2001), with cumulative distribution function (CDF) of Z_t (monthly maxima of the significant wave heights observed in month t) given by¶

$$F_t(z) = \begin{cases} \exp\left\{-\left[1 + \xi(t)\right]\right. \\ \left. \exp\left\{-\exp\left\{-\frac{z - \mu(t)}{\psi(t)}\right\}\right\}\right\} \end{cases}$$

where $[a]_+ = \max[a, 0]$.

The GEV distribution includes the three classical distribution families of extreme value theory: Gumbel family ($\xi = 0$); Fréchet distribution ($\xi > 0$), and Weibull family ($\xi < 0$).¶

Figure 2 shows the total population of SWH for each location and the monthly maxima sample. A clear seasonal variation is observed in all the points (stronger in north latitudes, North Point and Bretagne) and also a clear interannual variability can be appreciated, with severe and mild years, due to the natural climate variability. Since most of the variability is explained by seasonal behavior (Izaguirre et al. 2011) the introduction of harmonic functions to model seasonality is used (Menéndez et al., 2009). We let the model introduce the best number of harmonics in the three parameters.¶

On the other hand, the ... [1]

382 extreme wave climate in terms of severity. However, Coruña shows a more complex
383 parametrization due to the different sea families that arrive at this location in different
384 parts of the year.

385 In the right panels, the interannual variability of the time-dependent 50-year return
386 period quantile, δH_{50} , is presented. Note the variation of intensity between locations,
387 reaching 4.8 m of significant wave height anomaly in the North Point, while only 1.2 m
388 is reached in Azores. A variation in the intensity of the anomaly in every point is also
389 observed. The northern points have more marked interannual variability whilst Coruña
390 and Lisbon are less affected by the regional patterns of the North Atlantic having milder
391 interannual variability.

392 ~~SOMs have been applied for meteorological problems, for instance Cavazos~~
393 (2000) classifying climate modes, Gutierrez et al. (2005) analyzing multi-model
394 seasonal forecast, Cassano et al. (2006) classifying synoptic patterns in the western
395 Arctic or Reusch et al. (2007) classifying the North Atlantic climate variability.
396 Depending on the purpose of the work, the lattice size of the SOM is different. After
397 some preliminary tests, we have considered a SOM lattice of $8 \times 8 = 64$ groups, which
398 fulfils the compromise between a significant number of weather types and the
399 requirement of a minimum number of data per group.

400 Figure 4 shows the fields forming the atmospheric patterns for the resulting
401 reference vectors of the 8×8 SOM, the weather types of the North Atlantic. In this
402 figure one can see similar states close to each other and the most extreme states located
403 at the corners. The most common well-known patterns can be identifying in the grid.
404 The weather type located in the lower right corner corresponds to the synoptic situation
405 of the positive phase of the NAO, characterized by low pressures centered in the south
406 of Iceland and high pressures in the Azores Islands. The surrounding cells show

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407 transition states with variations of the synoptic pattern, till the negative phase, found
408 approximately in the middle rows of the left columns. On the other hand, the upper left
409 weather type shows a very different situation, characterized by positive anomaly of
410 pressure centered above the north-western part of Europe, similar to a blocking situation
411 describe by Cassou et al. (2011). The Atlantic Ridge weather regime describe in Cassou
412 et al. (2011) can be found in the upper right corner, and the positive phase of the East
413 Atlantic pattern (north-south dipole anomalies, similar to NAO but southerly shifted) in
414 the middle maps of the last row.

Eliminado: the opposite

Eliminado: high pressures

Eliminado: in Great Britain

415 The distribution from the high-dimensional space can be transformed into
416 probability density function on the SOM lattice. Each centroid, c_i , has a probability of
417 occurrence, p_i (figure 5), according to the histogram of winner clusters for each

418 atmospheric data, so that $\sum_{i=1}^N p_i = 1$, where N is the SOM size ($N = 64$ in this case).

419 The extreme wave climate can be projected similarly, representing in each cell the
420 average value of the corresponding MSLP dates within each cluster in order to establish
421 the relationship between the atmospheric conditions and extreme wave climate.

422 In this section, we establish a connection between the study of atmospheric
423 patterns in the North Atlantic and the extreme wave climate in different locations. Note
424 that the SOM technique, besides obtaining the most representative synoptic situations in
425 the NE Atlantic, also provides the possibility of representing a local climate variable at
426 a particular location on the SOM lattice by projecting the variable value associated to
427 each MSLP field. The process is summarized as follows:

428 1) We obtain synoptic atmospheric situations clustering standardized MSLP (in
429 terms of PCs) by using SOM pattern (figure 3).

430 2) For a specific location, using the standardized MSLP dates, we identify the
431 corresponding wave data of each cluster.

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Eliminado: for a specific location

432 3) For each cluster we calculate the monthly quantile anomalies of extreme SWH
433 (in terms of interannual variability) subtracting the seasonal-dependent quantile
434 from the time-dependent quantile.

435 4) We calculate the mean quantile anomaly of extreme SWH in each cluster,
436 together with its significance at 10 % level, and show the results in SOM-lattice
437 format (figure 6).

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438 Figure 6 shows the monthly extreme wave anomaly function on the SOM lattice
439 for each location. Significant values at 10 % level are represented with a dot in the
440 middle of the cell. One can see higher interannual variability in the northern points,

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441 reaching 2.5 m of positive anomaly in the North Point, while in Azores the higher
442 interannual anomaly reaches 1 m. This graphical representation, together with the
443 weather types, provide an easy way to identify atmospheric situations that produce

444 positive or negative anomalies (interannual variability) in the extreme wave climate at a
445 specific location. The anomaly in each cell is linked with its corresponding synoptic
446 pattern. Note that smooth variations of the quantile anomalies through the SOM lattice

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447 and clear groups of SOM states generate an increase or decrease in the H_{50} . For
448 instance, in the North Point the weather types located in the first columns, characterized

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Eliminado: decrement

449 by positive anomaly of pressure, centred above Iceland and northern Great Britain
450 generate negative anomalies in the extreme wave climate (up to -1.5 m). On the other
451 hand, during years characterized by atmospheric situations located in the last columns

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Eliminado: and low pressures above the Iberian Peninsula,

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452 of the lattice (positive phase of the NAO situation) the extreme wave climate in North
453 Point increases (positive anomaly reaching 2.5 m). These results are consistent with
454 those obtained in Wang and Swail (2002), where the winter seasonal 99th percentile of

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455 SWH in the North Atlantic is predicted by a NAO-like structure of SLP. Dodet et al.
 456 (2010) also showed high correlation between the NAO index and winter wave
 457 parameters, finding higher correlation at northern latitudes, north of 55°, where the
 458 North Point of this study is located. In the case of Bretagne, Landes and Coruña, the
 459 positive/negative extreme wave anomaly pattern in the SOM lattice is quite similar,
 460 varying slightly with respect to the intensity of the anomaly. The characteristic synoptic
 461 situation of positive phase of NAO (weather type in the lower right corner) generates
 462 positive anomalies of extreme wave height, especially in Bretagne and Landes (up to 2
 463 m). Besides, the states in the last row, representing the East Atlantic pattern, generate
 464 lower intensity of positive extreme wave height anomalies. Note that the positive phase
 465 of NAO accounts for increased storminess in the mid North Atlantic but also de East
 466 Atlantic pattern is an important factor for the storminess in the middle of the North
 467 Atlantic (Seierstad et al., 2007). It is also remarkable the positive extreme wave height
 468 anomaly generate by the Atlantic Ridge (weather type in the upper right corner). On the
 469 other hand, the negative anomalies are generated by the blocking situation and transition
 470 states (upper left corner and surroundings). Lisbon shows negative anomalies of
 471 extreme wave height (≈ -0.8 m) related to weather types similar to a blocking situation,
 472 characterized by positive anomaly of pressure centred over the north-western part of
 473 Europe, and the dipole of anomalies in the east-west direction. Finally, Azores shows a
 474 different positive/negative wave extreme anomaly pattern in the SOM lattice. In this
 475 case, the weather types located in the last column and first row generate the higher
 476 negative anomalies (up to -1 m). The weather types in the last column can be related to
 477 the positive phase of the NAO pattern and transition states. The ones in the first row are
 478 more similar to a blocking situation. On the contrary, weather types characterized by

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Eliminado: s located in the northeast of the North Atlantic basin. Most of them are transition states of the negative phase of the NAO situation.

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479 negative anomaly of pressure over the central North Atlantic generate an increment in
480 the extreme wave climate.

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481 In conclusion, it is remarkable that this technique allows identifying the synoptic
482 patterns responsible of an increase in the extreme wave height of a specific place. Note
483 that those synoptic patterns depend on the location of the studied point.

484

485 6. CONCLUSIONS

486 A methodological framework based on the SOM technique which provides a
487 simple graphical representation of the link between the interannual variability of
488 extreme wave climate with the synoptic patterns in the North Atlantic is presented.

489 The SOM classification is applied to principal components of monthly MSLP
490 anomalies to characterize a synoptic climatology of the North Atlantic area. The
491 resulting map shows patterns with variability in the Azores High and in the Icelandic
492 Low and smooth transitions between climate states.

493 On the other hand, a time-dependent GEV model including seasonal and
494 interannual variability is used to model the extreme wave height in six reanalysis
495 locations in the North Atlantic. Interannual variability is considered to depend on the
496 PCs of the monthly MSLP anomalies of the NA (Izaguirre et al. 2010). The best model
497 has been fitted to each reanalysis point. The annual cycle is observed in all locations,
498 with Coruña, Landes and Azores presenting the more complex parametrizations.

499 The 50-year return-period quantile anomalies for the studied locations have been
500 projected into the SOM lattice, obtaining maps that link the positive or negative
501 anomaly with the correspondent synoptic pattern. The projection of the extreme wave
502 climate allows comparing different severity between locations and identifying the most
503 energetic extreme wave families due to different atmospheric situations. Results show

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504 more influence of the interannual variability in the northern located points, where
505 synoptic patterns with a low pressure center near Iceland increase the 50-year return-
506 period quantile in the North Point by almost 2.5 m.

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507 The simplicity of evaluating the synoptic patterns using the SOM technique and
508 the representation of the consistent anomalies of extreme wave height in a certain
509 location on the synoptic SOM lattice, provide a useful and easy descriptive graphical
510 representation that helps understanding the effect of synoptic patterns at a global scale
511 on extreme wave climate at a regional scale.

512

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522

523 **REFERENCES**

524

525 Ancell R., and J.M. Gutiérrez, 2008: High resolution probabilistic forecast using
526 Bayesian networks. *EMS Annual Meeting 2008*. European Conference on Applied
527 Climatology (ECAC).

528

529
530
531
532
533
534
535
536
537
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541
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556
557
558
559
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561
562
563
564
565
566
567
568
569
570
571
572
573

Bacon and Carter, 1993: A connection between mean wave height and atmospheric pressure gradient in the North Atlantic. *Int. J. Climatol.*, **13**, 423-436.

Con formato: Fuente: 12 pt

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

Barry, R.G., and A.H. Perry, 1973: *Synoptic climatology: methods and applications*. Methuen & Co Ltd, London.

Eliminado: ¶

574 Bermejo M., and R. Ancell, 2009: Observed changes in extreme temperatures over
575 Spain during 1957-2002, using weather types. *Revista de Climatología*, **9**, 45-61.

576

577 Caires, S., V.R. Swail, and X.L. Wang, 2006: Projection and analysis of extreme wave
578 climate. *J. Climate*, **19(21)**, 5581-5605.

579

580 Cassano, E.N., A.H. Lynch, J.J. Cassano, and M.R. Koslow, 2006: Classification of
581 synoptic patterns in the western Arctic associated with extreme events at Barrow, Alaska,
582 USA. *Climate Research*, **30**, 83-97.

583

584 Cassou C, M. Minvielle, L. Terray, and C. Périgaud, 2011: A statistical-dynamical
585 scheme for reconstructing ocean forcing in the Atlantic. Part 1: weather regimes as
586 predictors for ocean surface variables. *Clim. Dyn.*, **36**, 19-39.

Con formato: Inglés (Reino Unido)

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

587

588 Cavazos, T., 2000: Using Self-Organizing Maps to investigate extreme climate event:
589 an application to wintertime precipitation in the Balkans. *J. Climate*, **13**, 1718-1732.

590

591 Coles, S. G., 2001: *An introduction to statistical modelling of extreme values*. London:
592 Springer, 208 pp.

593

594 Dodet, G., X. Bertin, and R. Taborda, 2010: Wave climate variability in the north-east
595 Atlantic Ocean over the last six decades. *Ocean Modell.*, **31**, 120-131.

Con formato: Inglés (Reino Unido)

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

Con formato: Inglés (Reino Unido)

596

597 Gutierrez J.M., R. Cano, A.S. Cofiño, and C.M. Sordo, 2004: Redes Probabilísticas y
598 Neuronales en las Ciencias Atmosféricas, *Series Monográficas de AEMET*.

599

600 Gutiérrez, J.M., R. Cano, A.S. Cofiño, and C. Sordo, 2005: Analysis and downscaling
601 multi-model seasonal forecast in Peru using Self-organizing Maps. *Tellus*, **57A**, 435-
602 447.

603

604 Hemer, M.A., 2010. Historical trends in Southern Ocean storminess: Long-term
605 variability of extreme wave heights at Cape Sorell, Tasmania. *Geophys. Res. Lett.*, **37**,
606 L18601.

607

608 Izaguirre, C., F.J. Méndez, M. Menéndez, A. Luceño, and I.J. Losada, 2010: Extreme
609 wave climate variability in Southern Europe using satellite data. *J. Geophys. Res.*, **115**,
610 C04009.

611

612 Izaguirre C., F.J. Méndez, M. Menéndez, and I.J. Losada, 2011: Global extreme wave
613 height variability based on satellite data. *Geophys. Res. Lett.*, **38**, L10607.

614

615 Jolliffe, I.T., 2002: *Principal Component Analysis* (2nd ed). Springer, New York.

616

617 Kalnay, E., M. Kanamitsu, R. Kistler, W. Collins, D. Deaven, L. Gandin, M. Iredell, S.
618 Saha, G. Walt, J. Woollen, Y. Zhu, M. Chelliah, W. Ebisuzaki, W. Higgins, J.
619 Janowiak, K.C. Mo, C. Ropelewski, J. Wang, A. Leetmaa, R. Reynolds, R. Jenne, and
620 D. Joseph, 1996: The NCEP/NCAR 40-year reanalysis project, *Bull. Amer. Meteor.*
621 *Soc.*, **77**, 437–471.

622

623 Kohonen, T., 1995: *Self-Organizing Maps*. Number 30 in Springer Series in Information
624 Sciences. Springer-Verlag.

625

626 Kohonen, T. 2001: *Self-Organizing Maps*. Springer-Verlag, Berlin, 3rd ed.

627

628 Kushnir, Y., V.J. Cardone, J.G. Greenwood, and M.A. Cane, 1997: The recent increase
629 in North Atlantic wave heights, *J. Climate*, **10**, 2107-2113.

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

630

631 Le Cozannet, G., S. Lecacheux, E. Delvallee, N. Desramaut, C. Oliveros and R.
632 Pedreros, 2011: Teleconnection Pattern Influence on Sea-Wave Climate in the Bay of
633 Biscay. *J. Climate*, **24**, 641-652.

634

635 Méndez, F.J., M. Menéndez, A. Luceño, and I.J. Losada, 2006: Estimation of the long-
636 term variability of extreme significant wave height using a time-dependent POT model.
637 *J. Geophys. Res.*, **111**, C07024.

638

639 Menéndez, M., F.J. Méndez, C. Izaguirre, A. Luceño, and I.J. Losada, 2009: The
640 influence of seasonality on estimating return values of significant wave height, *Coastal*
641 *Eng.*, **56(3)**, 211-219.

642

643 Mínguez, R., F.J. Méndez, C. Izaguirre, M. Menéndez, and I.J. Losada, 2010: Pseudo-
644 optimal parameter selection of non-stationary generalized extreme value models for
645 environmental variables. *Env. Mod. & Software*, **25**, 1592-1607.

646

647 Mínguez, R., A. Espejo, A. Tomás, F.J. Méndez, and I.J. Losada, 2011: Directional
648 Calibration of Wave Reanalysis Databases using Instrumental Data. *J. of Atmos.*
649 *Oceanic Technol.* (in press). doi: 10.1175/JTECH-D-11-00008.1.

650

651 Oja, E., and S. Kaski, 1999: Kohonen maps. *Elsevier*, Amsterdam.

652

653 Preisendorfer, R.W., and C.D. Mobley, 1988: *Principal component analysis in*
654 *meteorology and oceanography*. Developments in Atmospheric Science. Elsevier,
655 Amsterdam.

656

657 Reguero, B.G., M. Menéndez, F.J. Méndez, R. Mínguez, and I.J. Losada, 2012: A
658 global Ocean Wave (GOW) calibrated reanalysis from 1948 onwards. *Coastal*
659 *Engineering*, **65**, 38-55. Doi:10.1016/j.coastaleng.2012.03.003.

660

661 Reusch, D.B., R.B. Alley, and B.C. Hewitson, 2007: North Atlantic climate variability
662 from a self-organizing map perspective. *J. Geophys. Res.*, **112**, D02104.

663

664 Seierstad, I.A., D.B. Stephenson, and N.G. Kvamstø, 2007: How useful are
665 teleconnection patterns for explaining variability in extratropical storminess? *Tellus*,
666 **59A**, 170-181.

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

667

668 Smith, T.M., R.W. Reynolds, R.E. Livezey, and D.C. Stokes, 1996: Reconstruction of
669 historical sea surface temperatures using empirical orthogonal functions. *J. Climate*,
670 **9(6)**, 1403-1420.

671

672 Tolman, H.L., 2010: *WAVEWATCH III (R) development best practices Ver. 0.1*. NOAA
673 / NWS / NCEP / MMAB Technical Note 286, 19 pp.

674

675 Wang, X.L., and V.R. Swail, 2001: Changes of extreme wave height in northern
676 Hemisphere oceans and related atmospheric circulation regimes. *J. Climate*, **14**, 2204-
677 2221.

678
679 Wang, X.L., and V.R. Swail, 2002: Trends of Atlantic wave extremes as simulated in a
680 40-yr wave hindcast using kinematically reanalyzed wind fields. *J. Climate*, **15**, 1020-
681 1035.

Con formato: Fuente: Cursiva

Con formato: Fuente: Negrita

682
683 Woolf, D. K., P. G. Challenor, and P. D. Cotton, 2002: Variability and predictability of
684 the North Atlantic wave climate, *J. Geophys. Res.*, **107(C10)**, 3145.

Con formato: Inglés (Reino Unido)

685
686 **FIGURE CAPTIONS**

687
688 Figure 1. Spatial domain of the North Atlantic area and wave locations (NP, BR, LA,
689 CO, LI and AZ stands for North Point, Bretagne, Landes, Coruña, Lisbon and Azores,
690 respectively).

691
692 Figure 2. Time series of SWH (grey color) and monthly maxima (black line) for the six
693 analyzed locations.

694
695 Figure 3. Left panels: maximum SWH data (grey dots), location (grey line) and scale
696 (dashed grey line) parameters and 50-year return period quantile (black line). Right
697 panels: time series of the anomalies of the time-dependent 50-year return period
698 quantile (interannual variability).

699

700 Figure 4. Atmospheric synoptic patterns (using PCs of standardized MSLP derived from
701 NCEP-NCAR data between 1948-2008) in a SOM lattice of a 8 x 8.

702

703 Figure 5. Probability of occurrence of each weather type.

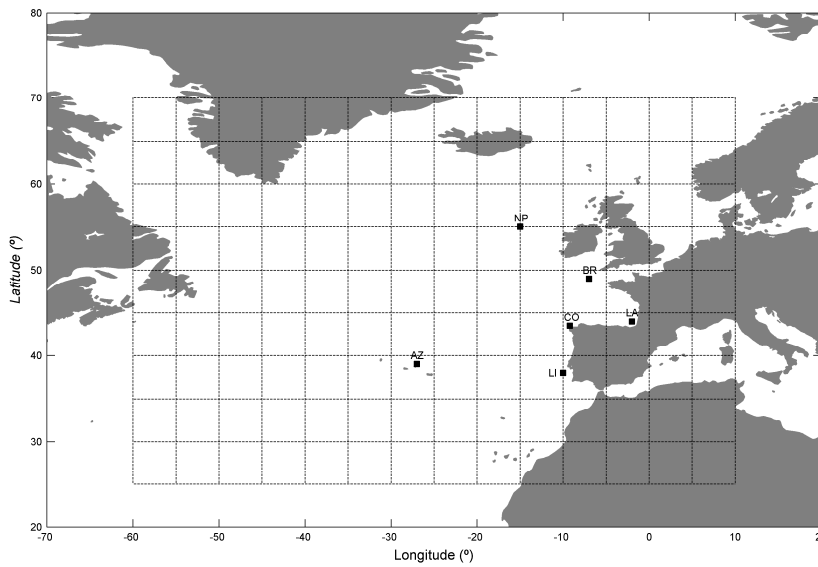
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705 Figure 6. Quantile anomalies (cm) associated to the 50-year return period projected in
706 the SOM lattice for the six reanalysis points. Significant values at the 90 % confidence
707 interval are dotted.

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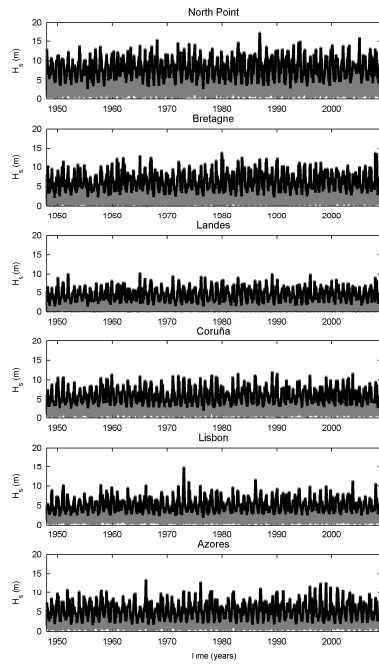
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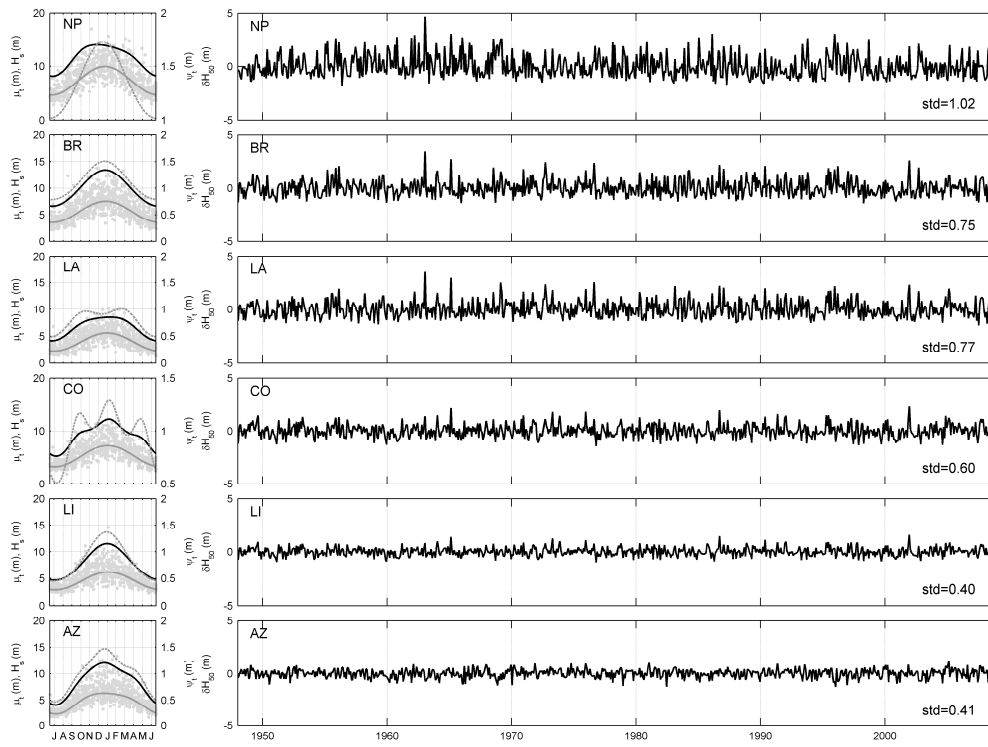
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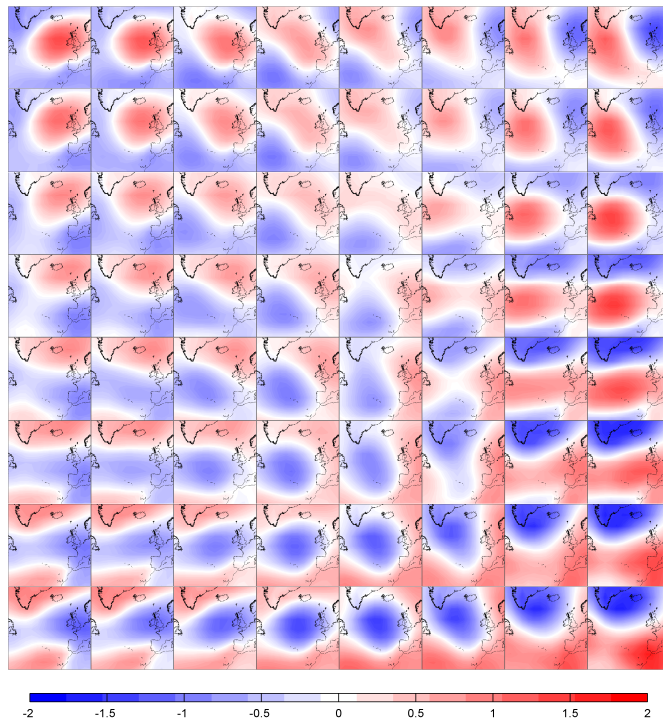
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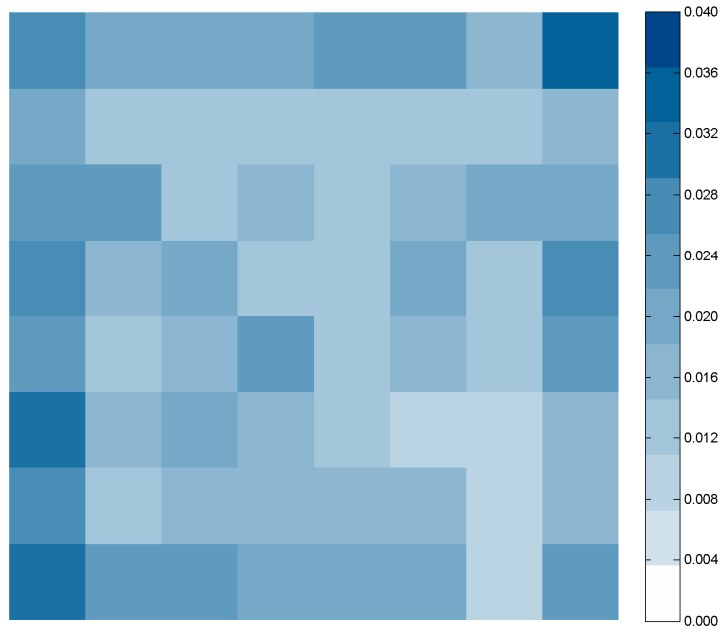
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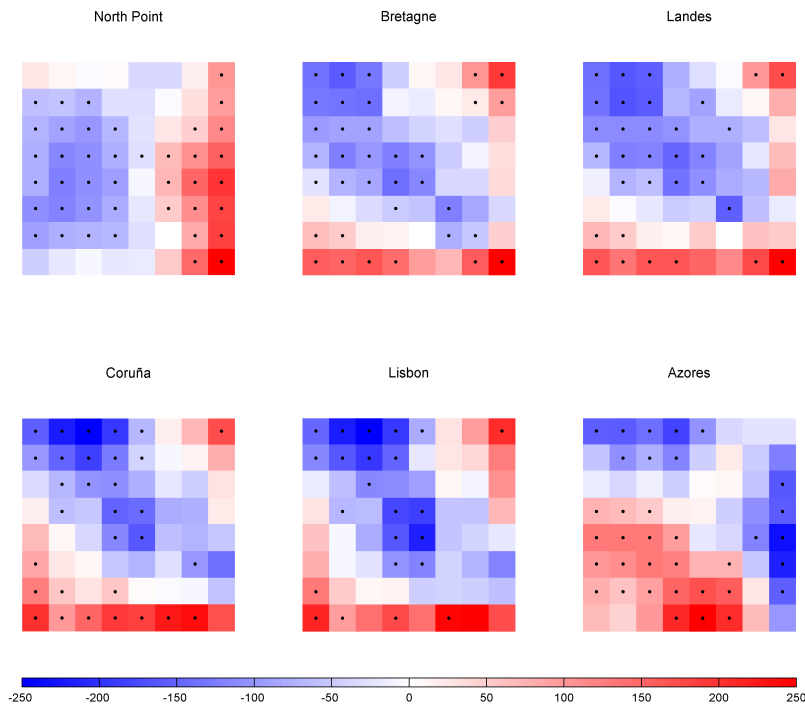
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3.3 Time-dependent extreme model

Latest advances in extreme value theory (see Coles 2001) allow a better description of the natural climate variability of extreme events of geophysical variables, specifically extreme wave height. In this work a time-dependent GEV model for monthly maxima SWH including seasonal and interannual variability is used. We have considered time-dependent location $\mu(t)$, scale $\psi(t) > 0$ and shape $\xi(t)$ parameters of the GEV (Coles 2001), with cumulative distribution function (CDF) of Z_t (monthly maxima of the significant wave heights observed in month t) given by

$$F_t(z) = \begin{cases} \exp \left\{ - \left[1 + \xi(t) \left(\frac{z - \mu(t)}{\psi(t)} \right) \right]_{+}^{-1/\xi(t)} \right\} & \xi(t) \neq 0 \\ \exp \left\{ - \exp \left[- \left(\frac{z - \mu(t)}{\psi(t)} \right) \right] \right\} & \xi(t) = 0 \end{cases}, \quad (5)$$

where $[a]_{+} = \max[a, 0]$. The GEV distribution includes the three classical distribution families of extreme value theory: Gumbel family ($\xi = 0$); Fréchet distribution ($\xi > 0$), and Weibull family ($\xi < 0$).

Figure 2 shows the total population of SWH for each location and the monthly maxima sample. A clear seasonal variation is observed in all the points (stronger in north latitudes, North Point and Bretagne) and also a clear interannual variability can be appreciated, with severe and mild years, due to the natural climate variability. Since most of the variability is explained by seasonal behavior (Izaguirre et al. 2011) the introduction of harmonic functions to model seasonality is used (Menéndez et al.,

2009). We let the model introduce the best number of harmonics in the three parameters.

On the other hand, the hypothesis that extreme wave climate is affected by regional SLP patterns is used and standardized PCs of monthly MSLP in the NE Atlantic are introduced as covariates to model interannual variability (Izaguirre et al., 2010). We let the model introduce up to ten PCs as linear terms in the location and scale parameter.

Mathematically, the model can be expressed as:

$$\mu(t) = \beta_0 + \sum_{i=1}^{P_\mu} [\beta_{2i-1} \cos(i\omega t) + \beta_{2i} \sin(i\omega t)] + \sum_{j=1}^{10} \beta_{PC_j} PC_j(t) \quad (6)$$

$$\log[\psi(t)] = \alpha_0 + \sum_{i=1}^{P_\psi} [\alpha_{2i-1} \cos(i\omega t) + \alpha_{2i} \sin(i\omega t)] + \sum_{j=1}^{10} \alpha_{PC_j} PC_j(t) \quad (7)$$

$$\xi(t) = \gamma_0 + \sum_{i=1}^{P_\xi} [\gamma_{2i-1} \cos(i\omega t) + \gamma_{2i} \sin(i\omega t)] \quad (8)$$

where β_0 , α_0 and γ_0 are mean values; β_i , α_i and $\gamma_i (i > 0)$ are the amplitudes of the harmonics; $\omega = 2\pi \text{ year}^{-1}$; P_μ , P_ψ , and P_ξ determine the number of sinusoidal harmonics in a year; and t is given in years. The parameter β_{PC_j} and α_{PC_j} represents the influence on the location and scale parameters per unit of standardized PC_j in a particular month, t . The model selection is carried out using the pseudo-optimal method explained in Minguez et al. (2010).

The instantaneous quantile $z_q(t)$ associated with the return period $1/q$ can be obtained using:

$$z_q(\mu(t), \psi(t), \xi(t)) = \begin{cases} \mu(t) - \frac{\psi(t)}{\xi(t)} \left[1 - \{-\log(1-q)\}^{-\xi(t)} \right] & \xi(t) \neq 0 \\ \mu(t) - \psi(t) \log\{-\log(1-q)\} & \xi(t) = 0 \end{cases} \quad (9)$$

where probability q is given by $F_t(z) = 1 - q$. Since seasonal and interannual variability have been modeled, the quantile varies depending on the time within the year and the year itself.

The interannual variation in the time-dependent quantile can be expressed as the difference between the time-dependent quantile (z_q) and the seasonal-dependent quantile (z_{qs}), where the seasonal-dependent quantile is calculated from a regression model where only the seasonal variation is considered.

$$\delta z_q = z_q - z_{qs} \quad (10)$$

where δz_q is the time-dependent quantile anomaly.