Design of a Composite Beam Using the Failure Probability-Safety Factor Method.

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Abstract

The paper aims to show the practical importance of the failure probability-safety factor method for designing engineering works. The method provides an automatic design tool by optimizing an objective function subject to two sets of constraints, that guarantee some given safety factors and failure probability bounds, associated with a given set of failure modes. Since a direct solution of the optimization problem is not possible, the method proceeds as a sequence of classical designs, based on given safety factors, that are modified for the design values to satisfy the safety factors and the failure probability constraints, until convergence. This implies a double safety check that leads to safer structures and designs less prone to wrong or unrealistic probability assumptions, and to too excessively small safety factors. Finally, the actual global or combined probabilities of the different failure modes are calculated using a Monte Carlo simulation. In addition, a sensitivity analysis is performed. To this end, the optimization problems are transformed into another equivalent ones, in which the data parameters are converted to artificial variables. In this way, some variables of the dual associated problems become the desired sensitivities. The method is illustrated by its application to the design of a composite beam.

Key Words: Sensitivity, Optimization, Automatic design.

1 Introduction and motivation

Engineering design of structural elements is a complicated and highly iterative process that usually requires a long experience. Iterations consists of a trial-and-error selection of the design variables or parameters, together with a check of the safety and functionality constraints, until a reasonable, in terms of cost and safety, structure is obtained.

Optimization procedures are a good solution to free the engineer from the above mentioned painful iterative process, i.e., to automate the design process. In this case, the values of the design variables are given by the optimization process and the engineer fixes only the constraints and the objective function to be optimized.

Note however, that iterations are then the responsibility of the optimization software that need to follow a numerical strategy to evolve from the initial to the final optimal design. It is worthy mentioning that the convergence properties of the optimization method are strongly dependent on the initial designs, i.e., a poor initial design can lead to unfeasibility problems, while a good initial design, leads to a very fast convergence. Thus, the importance of initial designs, where the participation of the engineer is unavoidable.

Several authors have used previously the optimization techniques to deal with engineering design, as for example, Lorenz [13], Kim and H. Adeli [12], Adeli [1], Bhatti [2], Sarma and Adeli

[20, 21], and Ringertz [17], etc. However, in this paper we deal with a new methodology that simultaneously considers safety factors and probabilities of failure.

Safety of structures is one of the fundamental criterion for design (see Eurocode [10], ROM [18],Freudenthal [11], Melchers [16], Wirsching and Wu [22], Wu, Burnside and Cruse [23]). To this end, the engineer lists first all failure modes of the work being designed and writes later the safety constraints to be satisfied by the design variables. To ensure satisfaction of the safety constraints, two apporoaches are normally used: (a) the classical approach, based on safety factors, and (b) the probability based approach, based on failure probabilities.

In the first case, the non-dimensional ratios of two opossing magnitudes (strengths to ultimate stresses, stabilizing to overturning loads, design to actual loads, etc.) are forced to exceed their corresponding safety factors, so that the failure is guaranteed to be far enough from occurrence.

In the second, the probabilities of failure of the different failure modes are forced to be below given reasonable bounds.

Since considered failure modes are coincident for both approaches, there is a, non-necessarily one-to-one, correspondence between the set of all safety factors and the sets of the corresponding failure probabilities, and this correspondence occurs through the values of the design variables, i.e., given any set of values for the design variables, the associated sets of safety factors and probabilities of failure can be calculated. Then, these two sets correspond to each other in such a correspondencee.

Note that the exhaustivity and the one-to-one character of this correspondence depends on the dimensions of the design variables space and the number of failure modes, the safety factor constraints and the failure probability constraints.

To avoid the lack of agreement between defenders of the classical and the probability based approaches, and to obtain a more reliable design, Castillo et al. [5] have proposed a mixed method that combines safety factors and failure probability constraints.

Since the failure probability bounds cannot be directly imposed in the form of standard constraints, optimization packages cannot deal directly with problems involving them. In fact, failure probability constraints require other optimization problems.

Fortunately, in Castillo et al. [5], an iterative method for solving these problems is given that converges in a few iterations to the optimal solution. The main advantage of this method is that the engineer is informed by the optimization method, not only on the values of the design variables leading to an optimal solution, but on the associated safety factors and failure probabilities for all failure modes. This implies a double safety check that leads to safer structures and designs less prone to wrong or unrealistic probability model, to tail distribution assumptions (see Castillo et al. [6, 7, 8]), and to too excessively small safety factors.

Note that in this way, the design is less dependent (more robust) with respect to the probability model assumptions, a criticism raised by defenders of the classical design.

In addition, the engineer would like to know how much changes in the data values alter the value of the objective function or the β-values (failure probabilities). This warns him/her from risky assumptions, points out the influence of the code constraints, and allows a better handling of the construction process if changes in the costs occur.

As we shall see, this sensitivity analysis can be easily performed, by free, with a single modification of the optimization problems involved.

In this paper, we discuss this procedure, and illustrate it by its application to the optimal design of a composite beam.

The paper is structured as follows. In section 2 we present the failure probability-safety factor (FPSF) method for designing of engineering works. In section 3 we show how to perform a sensitivity analysis of the design problem. In section 4 we describe the composite beam problem and solve it using the FPSF method. In section 5 a numerical example of the design is given and discussed. Finally, in section 6 we give some conclusions.

2 The probability of failure-safety factor method

As indicated in the introduction, the PFSF method minimizes

$$f(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0) \tag{1}$$

subject to

$$g_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0) \geq F_i^k;\ i \in I \tag{2}$$

$$h_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0) \leq P_i^k;\ i \in I \tag{3}$$

where $(x_1, x_2, \dots, x_n)$ are the values of the design variables $(X_1, X_2, \dots, X_n)$ to be determined by the optimization method, $\boldsymbol{\eta}_0$ is a vector of data, including the fixed variables or parameters, the cost of materials, the strength parameters (deterministic or statistic) of the materials involved, etc., $f(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ is the cost function, giving the total cost of the structure being designed, $g_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ are the non-dimensional ratios referred to at the introduction, $h_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ are the probabilities of failure, or their bounds, associated with the set of values $(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ for the design variables and data, I is the set of failure modes, $F_i^0; i \in I$ are the safety factors lower bounds, $P_i^0; i \in I$ are the failure probabilities upper bounds, and k refers to the kth iteration.

Since the constraints in (3) involve complicated integrals, a usual procedure to overcome this difficulty consists of transforming the initial set of random variables into a set of independent standard normal variables, for example, with the help of the Rosenblatt transformation [19]. Then, an upper bound for the i-th failure mode probability can be obtained by minimizing

$$\beta^2 = \sum_{i=1}^{n} z_i^2 \tag{4}$$

subject to

$$g_i^*(z_1, z_2, \dots, z_n; \boldsymbol{\eta}_0) = 1 \tag{5}$$

where $g_i^*(z_1, z_2, \dots, z_n; \boldsymbol{\eta}_0)$ is the function $g_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ after the above transformation.

Then, it can be easily shown (see Castillo et al. (2001)) that $\Phi(-\beta)$ can be taken as this upper bound. Then, constraints (3) can be replaced by

$$q_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0) \geq \beta_i^k;\ i \in I \tag{6}$$

where $\beta_i^{*k} = q_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}_0)$ is the optimal value of β after that transformation and optimization process.

For a complete description of some of these methods and some illustrative examples see Ditlevsen and Madsen [9] and Madsen, Krenk and Lind [15].

Based on this, Castillo et al [5] suggest the following algorithm.

Algorithm 1 (Design with safety factors and probability constraints)

- **Input:** The lower bounds $\{F_i^0 | i \in I\}$ for the safety factors $\{F_i | i \in I\}$, the upper bounds $\{\beta_i^0 | i \in I\}$ for the β-values $\{\beta_i | i \in I\}$, with respect to all failure modes, and an error value ϵ to control convergence of the procedure.
- **Output:** An optimal design defined by the values of the design variables, the actual safety factors, and the probabilities of failure associated with all different failure modes.

Initialization. Initialize all the safety factors to their lower bounds, and let $k = 0$.

Step 1. Determine an optimal classical design minimizing with respect to $\mathbf{x}$

$$f(x_1, x_2, \ldots, x_n; \boldsymbol{\eta}_0) \tag{7}$$

subject to

$$g_i(x_1, x_2, \ldots, x_n; \boldsymbol{\eta}_0) \geq F_i^k; \ i \in I \tag{8}$$

Step 2. Calculate exact values or lower bounds, $\beta_i^k; i \in I$, for the β-values for all failure modes, using a Level II or III approach. For example, minimizing with respect to $\mathbf{z}$

$$\beta = \sqrt{\sum_{j=1}^{n} z_j^2} \tag{9}$$

subject to

$$g_i^*(z_1, z_2, \ldots, z_n; \boldsymbol{\eta}_0) = 1, \tag{10}$$

Step 3. The safety factors are modified using the formula

$$\Delta F_i = \rho(\beta_i^0 - \beta_i^k); \ \ i \in I, \tag{11}$$

where $\beta_i^0; i \in I$ are the desired β lower bounds (associated with probability bounds), and ρ is a small positive constant.

To avoid large increments of the safety factors in each iteration, the value of ρ can be selected using the expression

$$\rho = \min\left(\rho_0, \min_i \left(\frac{\Delta}{|\beta_i^0 - \beta_i^k|}\right)\right), \tag{12}$$

where Δ is a small quantity, for example, $\Delta = 1$, and ρ_0 is a small number. In addition, if, using this formula, safety factor F_i^k becomes smaller than its associated lower bound F_i^0, it is kept equal to F_i^0.

In summary, the safety factors are updated using the formula

$$F_i^{k+1} = \max\left(F_i^0, F_i^k + \min\left(\rho_0, \min_i \left(\frac{\Delta}{|\beta_i^0 - \beta_i^k|}\right)\right)(\beta_i^0 - \beta_i^k)\right); \ i \in I \tag{13}$$

Step 4. If in the current iteration, changes in the design variables are larger than a given threshold value ϵ, let $k = k + 1$, and go to Step 1. Otherwise, go to Step 5.

Step 5. Calculate the actual safety factors associated with non-active constraints in (8).

Step 6. Stop and return design values, associated safety factors, and probabilities of failure for the different failure modes.

3 Sensitivity analysis

With the aim of performing a sensitivity analysis, the problem (7)-(8) can be modified to
Minimize with respect to $\mathbf{x}$ and $\boldsymbol{\eta}$

$$f(x_1, x_2, \dots, x_n; \boldsymbol{\eta}) \tag{14}$$

subject to

$$g_i(x_1, x_2, \dots, x_n; \boldsymbol{\eta}) \geq F_i^k;\ i \in I \tag{15}$$

and

$$\boldsymbol{\eta} = \boldsymbol{\eta}_0. \tag{16}$$

and the problem (9)-(10) can be modified to
Minimize with respect to $\mathbf{z}$ and $\boldsymbol{\eta}$

$$\beta = \sqrt{\sum_{j=1}^{n} z_j^2} \tag{17}$$

subject to

$$g_i^*(z_1, z_2, \dots, z_n; \boldsymbol{\eta}_0) = 1, \tag{18}$$

and

$$\boldsymbol{\eta} = \boldsymbol{\eta}_0. \tag{19}$$

Once this is done, the sensitivities of the cost function to the data are given by the values of the dual variables of the problem (14)-(16), and the sensitivities of the β-values to the data, by the values of the dual variables of the problem (17)-(19) (see, for example, Bazaraa et al. [3] and Luenberger [14]).

4 The composite beam problem

In this section we describe and design a composite beam (see Figure 1). Nevertheless, we clarify the reader that the aim of this example is to illustrate the PFSF method and not making an exhaustive enumeration and analysis of all possible failure modes and a rigurous analysis of the structural problem. Do not let the trees to hide the forest.

Composite beams are widely used in bridges, multistory buildings, commercial plants, etc. The main idea behind composite beams consists of replacing expensive steel material by cheap concrete material where compressions accur. To this end, a concrete deck is attached to the top of the compression flange of a steel beam using shear studs.

Design of composite beams usually involves selection of the following parameters: slab thickness, beam spacing, slab steel reinforcement, shear studs (number, size), etc.

4.1 Beam description

Consider the composite beam in Figure 2, that gives its geometry (note that it is parametrically defined).

To facilitate the beam description, we consider the following sets of indices:

e: Set of locations: {concrete slab (c), upper steel flange (u), lower steel flange (l)}.

t: Set of time periods: $\{0, \infty\}$.

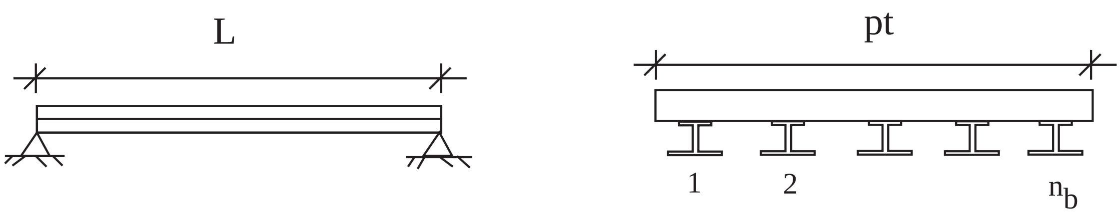

Figure 1: Composite beam used in the illustrative example.

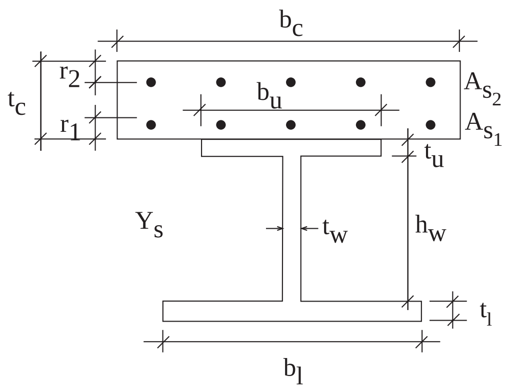

Figure 2: Geometrical definition of the composite beam used in the illustrative example.

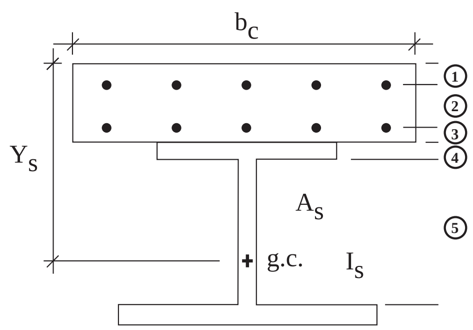

Figure 3: Different regions of the plastic neutral axis.

p: Set of cross sections positions: {beam center (c), support (s)}.

n: Set of plastic neutral axis level: $\{1, 2, 3, 4, 5\}$ (see Figure 3).

The main elements used in the composite beam design are:

1. **Data**

   (a) **Design constants**

   L: bridge span (m).

   pl: plattform width (m).

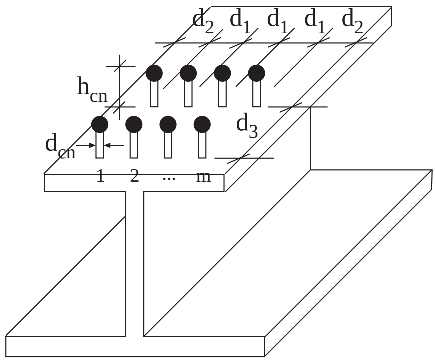

Figure 4: Detail of the connectors and their spacings.

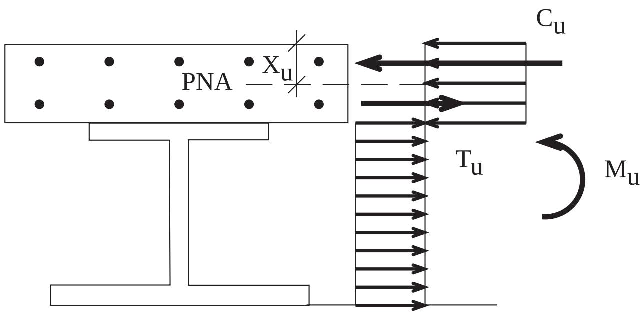

Figure 5: Illustration of the flexure.

d_{st}: shear stud diameter (m).

r_1: lower concrete passive reinforcement covering (m).

r_2: upper concrete passive reinforcement covering (m).

n_b: number of steel beams.

m: number of shear studs per cross section (m).

(b) **Material properties**

γ_c: concrete unit weight (kN/m^3).

γ_s: steel unit weight (kN/m^3).

f_{sk}: elastic limit of passive steel (MPa).

f_{yk}: elastic limit of structural steel (MPa).

E_t: concrete Young modulus at time $t \in \{0, \infty\}$ (MPa).

E_s: steel's Young modulus.

ϕ: concrete fluence coefficient.

(c) **Statistical properties**

μ_{w_d}: mean value of w_d.

μ_{w_p}: mean value of w_p.

μ_{p_s}: mean value of p_s.

v_{w_d}: coefficient of variation of w_d.
v_{w_p}: coefficient of variation of w_p.
v_{p_s}: coefficient of variation of p_s.
vf_c: coefficient of variation of f_c.
vf_s: coefficient of variation of f_s.
vf_y: coefficient of variation of f_y.
f_{ck}: characteristic concrete compressive strength (MPa).
w_{pk}: characteristic moving cart-load (KN).
w_{dk}: characteristic dead load (KN/m).
p_{sk}: characteristic surcharge per unit surface load (kN/m^2).

(d) **Load data**

e_p: point load excentricity coefficient (nondimensional).
e_s: surcharge excentricity coefficient (nondimensional).

(e) **Mode safety factors lower bounds.**

F^0_{c0}: safety factor for the concrete capacity in the center of the beam at time 0.
$F^0_{c\infty}$: safety factor for the concrete capacity in the center of the beam at time ∞.
F^0_{u0}: safety factor for the steel upper flange capacity in the center of the beam at time 0.
$F^0_{u\infty}$: safety factor for the steel upper flange capacity at the center of the beam at time ∞.
$F^0_{\ell 0}$: safety factor for the steel lower flange capacity at the center of the beam at time 0.
$F^0_{\ell\infty}$: safety factor for the steel lower flange capacity at the center of the beam at time ∞.
F^0_w: safety factor for the steel web capacity.
F^0_m: safety factor for the ultimate moment capacity of the beam.
F^0_v: safety factor for the ultimate shear capacity of the beam.

(f) **Cost data**

c_c: cost per cubic meter of concrete (euros/m^3).
c_s: cost per Newton of passive steel (euros/N).
c_y: cost per Newton of structural steel (euros/N).
c_{st}: cost per shear stud (euros).

2. **Variables**

(a) **Design variables**

b_e: width of element $e \in \{c, u, \ell\}$ (m).
t_e: thickness or height of element $e \in \{c, u, \ell\}$ (m).
h_w: steel web height (m).
t_w: steel web thickness (m).
d_n: connector separation $n \in \{1, 2, 3\}$ (m).
A_{s_1}: slab lower concrete passive reinforcement (m).
A_{s_2}: slab upper concrete passive reinforcement (m).
h_{st}: shear stud height (m).

(b) **Random variables**

f_c: concrete strength used in calculations (MPa).

f_s: passive steel strength used in calculations (MPa).

f_y: structural steel strength used in calculations (MPa).

w_p: moving cart point load (KN).

w_d: dead load acting on all plattform width (KN/m).

p_s: surcharge per unit surface (KN/m^2).

(c) **Auxiliary or intermediate variables**

w_1: beam total weight per unit length (KN/m).

l_d: dead load acting on all beam width (KN/m).

C_p: point load acting on the beam (KN).

s_u: surcharge acting on the beam per unit length (KN/m).

M_p: moment at position $p \in \{c, s\}$ (MN.m).

V_t: position of the cross section center of gravity at time $t \in \{0, \infty\}$ (m).

V_p: shear force at position $p \in \{c, s\}$ (KN).

$A_{e,t}$: concrete slab cross section at location $e \in \{c, u, \ell\}$ and time $t \in \{0, \infty\}$. (m^2).

A_w: steel web cross section (m^2).

I_t: concrete slab cross section moment of inertia at time $t \in \{0, \infty\}$ (m^4).

I_s: steel web cross section moment of inertia (m^4).

Y_e: coordinate of the concrete slab center of gravity at position $e \in \{c, u, \ell\}$ (m).

Y_w: location of the steel web cross section center of gravity (m).

EI_t: concrete siffness at time $t \in \{0, \infty\}$ (MNm^2).

δ_t: beam deflection in the center of the beam at time $t \in \{0, \infty\}$ (m).

δ: beam deflection in the center of the beam at time ∞ after subtracting deflection at time 0 (m).

A_{min}: minimum passive steel reinforcement (m^2).

e_{max}: maximum thickness of steel slabs (m).

$\sigma_{e,t}$: normal stress at location $e \in \{c, u, \ell\}$ and time $t \in \{0, \infty\}$ (MPa).

τ_p: web tangential stress at position $p \in \{c, s\}$ (MPa).

C_{un}: compression block when the assumed plastic neutral axis is at level $n \in \{1, 2, 3, 4, 5\}$ (MN).

T_{un}: tensile block when the assumed plastic neutral axis is at level $n \in \{1, 2, 3, 4, 5\}$ (MN).

X_{un}: actual plastic neutral axis position when the assumed plastic neutral axis is at level $n \in \{1, 2, 3, 4, 5\}$ (m).

M_{un}: ultimate moment capacity when the assumed plastic neutral axis is at level $n \in \{1, 2, 3, 4, 5\}$ (MN).

τ_{cr}: maximum tangential stress supported by the web (MPa).

λ_w: auxiliary coefficient for calculting the maximum shear force supported by the web.

m_t: auxiliary coefficient for calculting the maximum shear force supported by the web.

τ_{max}: maximum tangencial stress supported by the structural steel (MN/m^2).

V_{max}: maximum shear force supported by the structural steel (MN/m^2).

P_{stc}: maximum force supported by the shear stud surrounding concrete (MN).

P_{st}: maximum force supported by the shear studs (MN).
N_{cs}: maximum force supported by the concrete slab (MN).

3. **Constraints.** These are the constraints associated with the different modes of failure.
   (a) failure of the concrete at time 0.
   (b) failure of the concrete at time ∞.
   (c) failure of the steel upper flange at time 0.
   (d) failure of the steel upper flange at time ∞.
   (e) failure of the steel lower flange at time 0.
   (f) failure of the steel lower flange at time ∞.
   (g) failure of the steel web.
   (h) failure due to the ultimate moment capacity of the beam.
   (i) failure due the ultimate shear force capacity of the beam.
4. **Function to be optimized.** We optimize the cost function, i.e., we minimize the composite beam cost per unit length.

4.2 Geometric and mechanical properties of the beam elements

Cross sections (measured as steel equivalent sections):

$$A_{c,t} = \frac{b_c t_c E_t}{E_s} \tag{20}$$
$$A_{u,t} = b_u t_u \tag{21}$$
$$A_{\ell,t} = b_\ell t_\ell \tag{22}$$
$$A_w = h_w t_w \tag{23}$$

Coordinates of the center of gravity for the different pieces:

$$Y_c = t_c/2 \tag{24}$$
$$Y_u = t_c + t_u/2 \tag{25}$$
$$Y_\ell = t_c + t_u + h_w + t_\ell/2 \tag{26}$$
$$Y_w = t_c + t_u + h_w/2 \tag{27}$$

Moments of inertia (measured as steel equivalent sections):

$$I_t = \frac{b_c t_c^3 E_t}{12 E_s} \tag{28}$$
$$I_s = \frac{b_u t_u^3}{12} + \frac{b_\ell t_\ell^3}{12} + \frac{t_w h_w^3}{12} \tag{29}$$

Center of gravity of the cross section including all materials:

$$V_t = \frac{\sum_e A_{e,t} Y_e + A_w Y_w}{\sum_e A_{e,t} + A_w} \tag{30}$$

Stiffness:

$$EI_t = E_s\left(I_t + I_s + \sum_e A_{e,t}(Y_e - V_t)^2 + A_w(Y_w - V_t)^2\right) \quad (31)$$

Deflections:

$$\delta = \delta_\infty - \delta_0 \quad (32)$$

Concrete Young modulus at different times:

$$E_0 = 10000\sqrt[3]{f_c} \quad (33)$$

$$E_\infty = \frac{E_0}{1+\phi} \quad (34)$$

4.3 Actions on the beam

The weight per unit length corresponding to a single beam is

$$w_1 = \gamma_c b_c t_c + \gamma_s\left(b_u t_u + b_\ell t_\ell + h_w t_w\right) \quad (35)$$

and the proportional part of the dead load is

$$l_d = \frac{w_d b_c}{p_\ell} \quad (36)$$

The proportional part of the point load per beam is

$$C_p = \frac{w_p b_c e_p}{p_\ell} \quad (37)$$

and the proportional part of the surcharge considering an excentricity amplification factor to take into account the existence of n_b beams becomes

$$s_u = p_s b_c e_s \quad (38)$$

Then, the resulting bending moments and shear forces at the center of the beam and supports are

$$M_c = \frac{(w_1 + l_d + s_u)L^2}{8000} + \frac{C_p L}{4000} \quad (39)$$

$$M_s = 0 \quad (40)$$

$$V_c = \frac{C_p}{2000} \quad (41)$$

$$V_s = \frac{(w_1 + l_d + s_u)L}{2000} + \frac{C_p L}{1000} \quad (42)$$

4.4 Safety factor constraints

We start by writing the safety factors constraints associated with all failure modes.

Concrete capacity at times 0 and ∞ failure modes. The analysis of these failures can be done by considering the ratio of the strength to actual stresses as

$$\frac{f_c}{\sigma_{c,t}} = \frac{f_c}{\dfrac{M_c V_t E_t}{EI_t}} \geq F_{ct};\ t \in \{0, \infty\} \tag{43}$$

where F_{c0} and $F_{c\infty}$ are the safety coefficient associated with two these failure modes.

Steel upper and lower flange capacity at times 0 and ∞ failure modes. The analysis of this failure can be done by considering the ratio of the strength to actual stresses as

$$\frac{f_y}{\sqrt{\sigma_{u,t}^2 + 2\tau_c^2}} = \frac{f_y}{\sqrt{\left(\dfrac{M_c(V_t - t_c)E_s}{EI_t}\right)^2 + 3\left(\dfrac{V_t}{h_w t_w}\right)^2}} \geq F_{u,t};\ t \in \{0, \infty\} \tag{44}$$

and

$$\frac{f_y}{\sqrt{\sigma_{\ell,t}^2 + 2\tau_c^2}} = \frac{f_y}{\sqrt{\left(\dfrac{M_c(t_c + t_w + h_w + t_i - V_t)E_s}{EI_t}\right)^2 + 3\left(\dfrac{V_t}{h_w t_w}\right)^2}} \geq F_{\ell,t};\ t \in \{0, \infty\} \tag{45}$$

where $F_{u,0}, F_{u,\infty}, F_{\ell,0}$ and $F_{\ell,\infty}$ are the safety coefficient associated with these plastic failure modes.

Steel web capacity failure mode. The analysis of this failure can be done by considering the ratio of the shear steel strength capacity to the actual shear force as

$$\frac{f_y}{\sqrt{3\tau_s^2}} = \frac{f_y}{\dfrac{V_s\sqrt{3}}{h_w t_w}} \geq F_w \tag{46}$$

where F_w is the safety coefficient associated with this failure mode.

Ultimate bending moment capacity at the center of the beam failure mode. The analysis of this failure can be done by considering the ratio of the resistant moment to the actual bending moment as

$$\frac{M_u}{M_c} \geq F_m \tag{47}$$

where F_m is the safety coefficient associated with this failure mode (see Figure 6).

Ultimate shear force capacity at the buttress of the beam failure mode. The analysis of this failure can be done by considering the ratio of the shear concrete strength capacity to the actual shear force as

$$\frac{V_{max}}{V_s} \geq F_v \tag{48}$$

where F_v is the safety coefficients associated with this failure mode.

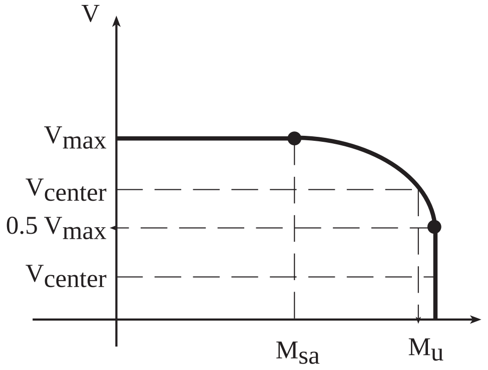

Figure 6: Illustration of the interaction shear-deflection.

Since, including the three flexural and three shear, we have 9 different failure modes, from now on, we define the set I_f of failure modes as

$$I_f = \{c0, c\infty, u0, u\infty, \ell 0, \ell\infty, w, mc, vs\}$$

The beam will be safe if and only if $F_{c0}, F_{c\infty}, F_{u0}, F_{u\infty}, F_{\ell 0}, F_{\ell\infty}, F_w, F_m, F_v \geq 1$.

Note that only the usually relevant failure modes have been considered, but other failures are also possible, as shallow or deep soil failures; however, for the sake of clarity, they have been ignored in this paper.

It is important to mention that we consider not the serviceability states, but the ultimate limit states.

4.5 Various constraints

We assume that the concrete works at compression:

$$V_t \geq t_c; \; t \in \{0, \infty\} \tag{49}$$

We assume a non slender bridge, i.e., the following constraint is satisfied

$$\frac{pl}{n_b} \leq 0.1L \tag{50}$$

4.6 Design criteria and code requirements

The following constraints are geometrical ,constraints fixed by the Spanish code:

$$\begin{aligned}
0.008 &\leq t_u \leq e_{max} && (51)\\
0.008 &\leq t_\ell \leq e_{max} && (52)\\
0.008 &\leq t_w \leq e_{max} && (53)\\
0.180 &\leq t_c && (54)\\
b_c/4 &\leq b_u && (55)\\
b_c &= pl/n_b && (56)\\
b_\ell &\leq 30t_\ell && (57)
\end{aligned}$$

The minimum steel required amount is

$$A_{s_1} \geq 0.0009 b_c t_c \text{ and } A_{s_2} \geq 0.0009 b_c t_c \tag{58}$$

According to the Spanish Code, the following constraint in the steel height must be satisfied

$$e_{max} = 0.150\text{m} \quad \text{if} \quad f_{yk} = 235MPa \tag{59}$$
$$e_{max} = 0.084\text{m} \quad \text{if} \quad f_{yk} = 275MPa \tag{60}$$
$$e_{max} = 0.038\text{m} \quad \text{if} \quad f_{yk} = 355MPa \tag{61}$$

and the following maximum deflection at the center of the beam is allowed

$$\delta \leq \frac{L}{800} \tag{62}$$

where $\delta = \delta_\infty - \delta_0$, i.e., the deflection at time ∞ measured with respect to the deflection at time 0, and

$$\delta_\infty = 2\int_0^{L/2} \frac{(w_1 + l_d + s_u)(L-x)x^2 + C_p x^2}{4000EI_\infty} dx \tag{63}$$

and

$$\delta_0 = 2\int_0^{L/2} \frac{(w_1 + l_d)(L-x)x^2}{4000EI_0} dx \tag{64}$$

Limit state of web deformations:
To avoid the web bump the following conditions must be satisfied:

$$\frac{h_w}{t_w} \leq 0.55\frac{E_s}{f_y}\sqrt{K} \tag{65}$$
$$\frac{h_w}{t_w} \leq 100\sqrt{\frac{355}{f_y}} \tag{66}$$

where $K = \dfrac{h_w t_w}{b_u t_u}$.

In addition we must have

$$f_c = f_{ck} + 8 \tag{67}$$

In this example we do not consider the vibration problem of the beam.

4.7 Connector constraints

We also use the following geometric connector constraints:

$$4d_{st} \leq h_{st} \leq 0.75t_c \tag{68}$$
$$d_3 \geq 5d_{st} \tag{69}$$
$$2.5d_{st} \leq d_1 \leq 0.8\text{m} \tag{70}$$
$$d_2 = 0.05\text{m} \tag{71}$$
$$d_3 \leq 0.8\text{m} \tag{72}$$
$$d_{st} \leq 2.5t_u \tag{73}$$
$$d_1(m-1) = b_u - 2d_2 \tag{74}$$
$$d_3 \geq 0.1\text{m} \tag{75}$$

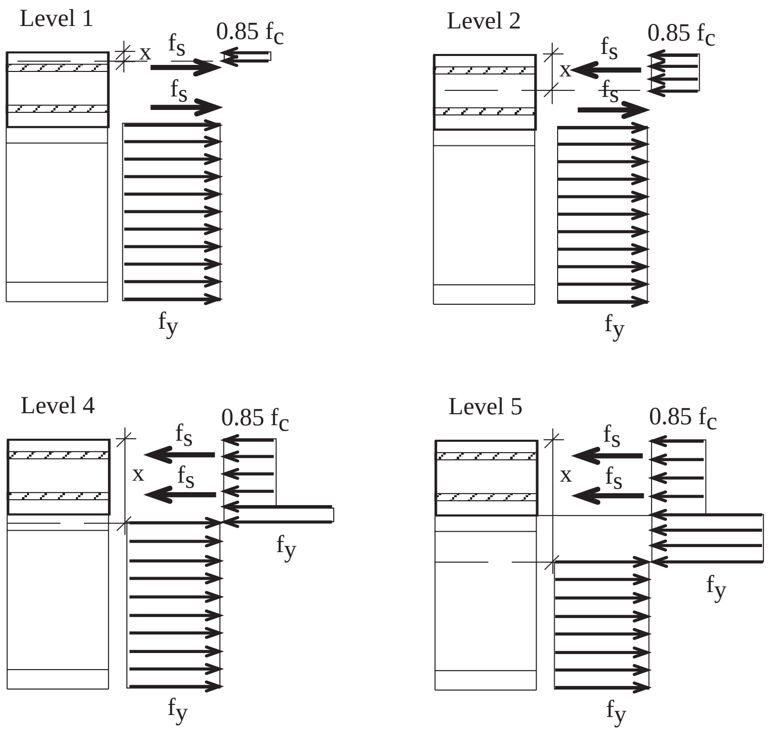

Figure 7: Possible positions of the plastic neutral axis.

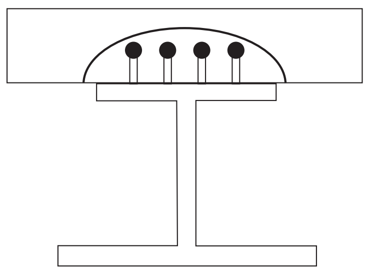

Figure 8: Illustration of a connector failure.

To avoid the connector failure due to failure of the surrounding concrete we must have (see Figure 8).

$$0.6 \times 0.29 \frac{d_{st}^2}{1.25} \sqrt{f_c E_c} \geq N_{cs} d_3/(mL/2) \tag{76}$$

and to avoid the direct connector failures

$$0.8 \times 450\pi \frac{d_{st}^2}{4 \times 1.25} \geq N_{cs} d_3/(mL/2) \tag{77}$$

where

$$N_{cs} = 0.85 f_c b_c t_c + (A_{s1} + A_{s_2}) f_s \tag{78}$$

4.8 Maximum shear constraints

$$\begin{aligned}
\tau_{cr} &= 5.34E_s 0.9(t_w/h_w)^2 && (79)\\
\lambda_w &= \sqrt{\frac{f_y}{\sqrt{3}\tau_{cr}}} && (80)\\
m_t &= \exp(-0.227086\lambda_w^2) && (81)\\
\tau_{max} &= \frac{m_t f_y}{\sqrt{3}} && (82)\\
V_{max} &= \tau_{max} h_w t_w && (83)\\
V_t &\leq V_{max}/2 && (84)
\end{aligned}$$

4.9 Function to be optimized

In this section we calculate the total cost of the composite beam. Since this is only an illustrative example of the proposed method, and for the sake of simplicity, we do not consider the life span of the composite beam, maintenance or reparation costs.

To this end, we need calculating the required total weight of concrete,, steel, and the number of connectors per unit length.

$$Cost = 1000c_y(\gamma_s b_u t_u + b_\ell t_\ell + h_w t_w) + c_c b_c t_c + 1000c_s\gamma_s(A_{s_1} + A_{s_2}) + c_{st}\frac{m}{d_3} \qquad (85)$$

4.10 Classical design of the composite beam

In a classical design of the composite beam the safety factors are given and we

$$\text{Minimize } 1000c_y(\gamma_s b_u t_u + b_\ell t_\ell + h_w t_w) + c_c b_c t_c + 1000c_s\gamma_s(A_{s_1} + A_{s_2}) + c_{st}\frac{m}{d_3}$$

subject to the safety constraints

$$\begin{aligned}
h_1(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{c0} && (86)\\
h_2(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{c\infty} && (87)\\
h_3(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{u0} && (88)\\
h_4(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{u\infty} && (89)\\
h_5(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{\ell 0} && (90)\\
h_6(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_{\ell\infty} && (91)\\
h_7(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_w && (92)\\
h_8(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_m && (93)\\
h_9(b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1, d_3; \boldsymbol{\eta}_0) &\leq F_v && (94)
\end{aligned}$$

and the remaining constraints.

Note that the other variables involved are intermediate variables, i.e., they are auxiliary variables that can be eliminated from the set of equations from (20) to (84). However, this is not necessary, and even convenient not to do so, to reduce the required work.

4.11 Designing at levels II and III

A composite beam design at levels II and III imply defining the random properties of all variables involved.

4.11.1 Distributional assumptions of the model

As it has been indicated, some variables are assumed to be independent random variables, with the following distributional assumptions for the following parameters:

1. **Geometric parameters.** All the design variables $b_c, b_u, b_\ell, t_c, t_u, t_\ell, b_w, t_w, A_{s_1}, A_{s_2}, d_1$ and d_3) are assumed to be deterministic, i.e. their values coincide with the design values with probability one.

2. **Mechanical material properties** The concrete strength, f_c, and the steel strengths, f_s and f_y, are assumed to be normal random variables. Their means arfe evaluated using the characteristic values f_{ck}, f_{sk} and f_{yk}, respectively, (there is a probability 0.95 that the real value is greater than the characteristics values:

$$\mu_{f_c} = \frac{f_{ck}}{1 - 1.64 v_{f_c}}.$$

$$\mu_{f_s} = \frac{f_{sk}}{1 - 1.64 v_{f_s}}.$$

$$\mu_{f_y} = \frac{f_{yk}}{1 - 1.64 v_{f_y}}.$$

The variables w_d, w_p and p_s are assumed to be independent random normal variables with given means μ_{w_d}, μ_{w_p} and μ_{p_s}, and coefficients of variation v_{w_d}, v_{w_p} and v_{p_s}, respectively.

4.11.2 Satisfying the failure probability constraints

Once the cost has been minimized subject to all the constraints, with the safety factors

$$\{F_{c0}, F_{c\infty}, F_{u0}, F_{u\infty}, F_{\ell 0}, F_{\ell\infty}, F_w, F_m, F_v\},$$

the corresponding bounds for the probabilities of failure

$$\{P_{c0}, P_{c\infty}, P_{u0}, P_{u\infty}, P_{\ell 0}, P_{\ell\infty}, P_w, P_m, P_v\}$$

or

$$\{\beta_{c0}, \beta_{c\infty}, \beta_{u0}, \beta_{u\infty}, \beta_{\ell 0}, \beta_{\ell\infty}, \beta_w, \beta_m, \beta_v\}$$

are calculated, and the safety factors are modified using the formula (11).

$$\Delta F_i^k = \rho(\beta_i^0 - \beta_i^k); \;\; i \in I_f, \tag{95}$$

where β_i^0 is the β-value associated with the corresponding probability upper bound, P_i^0, and ρ is a small positive constant.

Note that the safety factor has to be increased or decreased, depending on the difference between the associated actual and the desired β-values (failure probabilities). Since an increase of the safety factor decreases the corresponding failure probability and increases the β-values, the safety factor increment is made proportional to the difference $\beta_i^0 - \beta_i^k$.

Finally, if the resulting $F_i^k = F_i^{k-1} + \Delta F_i$ is smaller than the corresponding bound F_i^0, we make $F_i^k = F_i^0$, in order to satisfy the safety factors constraints.

5 A numerical example

The proposed method has been implemented in GAMS (General Algebraic Modeling System) (see Castillo, Conejo, Pedregal, García and Alguacil [4]) for a concrete example. Suppose we want to design the following composite beam:

$L = 30m$	$pl = 10m$	$e_p = 2;$	$e_s = 1.7;$
$E_s = 210000MPa;$	$\phi = 2;$	$\gamma_c = 25KN/m^3;$	$\gamma_s = 78.5KN/m^3;$
$m = 4;$	$d_{st} = 0.02m;$	$d_2 = 0.05m$	$n_b = 5$
$\mu_{p_s} = 4KN/m^2$	$\mu_{w_d} = 40KN/m$	$\mu_{w_p} = 600KN$	$v_{ps} = 0.3$
$v_{wd} = 0.2$	$v_{w_p} = 0.3$	$v_{f_y} = 0.03$	$v_{f_s} = 0.04$
$r_1 = 0.05m$	$r_2 = 0.05m$	$c_y = 0.2108euros/N$	$c_s = 0.062euros/$
$c_{st} = 2euros$	$c_c = 60.24euros/m^3$	$f_{ck} = 30MPa$	$f_{sk} = 400MPa$
$f_{yk} = 235MPa$			

where these are the data values that have been chosen by the designer.

Table 1 shows the convergence of the process that is attained after 5 iterations. The last column of the table shows the design values of the design variables $b_c, b_u, b_\ell, t_c, t_u, t_\ell, h_w, t_w, h_{st}, A_{s_1}, A_{s_2}, d_1$, d_2 and d_3, together with the safety factors and associated β-values. The design was done for guaranteeing values of the safety factors

$$F_{c0} = F_{c\infty} = 2.25; , F_{u0} = F_{u\infty} = 1.65; F_{\ell 0} = F_{\ell\infty} = 1.3; F_w = 1.65; F_m = F_v = 2$$

and

$$\beta_{cc0} = \beta_{cc\infty} = \beta_{cu0} = \beta_{cu\infty} = \beta_{cl0} = \beta_{cl\infty} = \beta_w = \beta_{mc} = \beta_{vs} = 3.719$$

corresponding to a failure probability of 10^{-4}.

The active values appear underlined in this table.

The following conclusions can be drawn from Table 1.

1. The process converges in only 9 iterations.

2. The list of actual safety factors and failure probabilities are obtained.

From this table we can get the following conclusions:

1. No safety factors are active, i.e., the maximum deflection constraint is as strict that none of the safety factor constraints are active.

2. Due to the terribly strict constraint imposed by the maximum deflection, only the probability bound $\beta_{F_{\ell\infty}}$ is active, and this is so, because it takes on a very high value 5.

5.1 Sensitivity analysis

The sensitivities for the composite beam example are given in Tables 2 and 3. Table 2 gives the cost sensitivities associated with the optimal classical design. It allows to know how much changes the total cost of the composite beam when a small change in a single data value is made. This information is extremely useful during the construction process to control the cost, and for analyzing how the changes in the safety factors required by the codes influence the total cost of engineering works. For example, a change of one euro in the unit cost c_s of the steel leads to a

Table 1: Illustration of the iterative procedure. The design and final values are bolfaced.

		ITERATIONS			
Variable	Units	1	2	3	9 (end)
Cost	*euros*	756.3	769.4	776.0	**783.3**
b_c	m	2.00	2.00	2.00	**2.00**
b_u	m	0.50	0.50	0.50	**0.50**
b_l	m	0.09	0.13	0.64	**0.67**
t_c	cm	33.27	33.27	33.27	**33.27**
t_u	cm	0.80	0.80	0.80	**0.80**
t_ℓ	cm	15.00	11.86	2.13	**2.22**
h_w	m	1.65	1.62	1.68	**1.66**
t_w	cm	1.37	1.35	1.41	**1.38**
h_{st}	cm	8.00	8.00	8.00	**8.00**
As_1	cm^2	5.99	5.99	5.99	**5.99**
As_2	cm^2	5.99	5.99	5.99	**5.99**
d_1	cm	13.33	13.33	13.33	**13.33**
d_2	cm	5.00	5.00	5.00	**5.00**
d_3	cm	17.16	17.16	17.16	**17.16**
F_{c0}	–	4.49	4.48	4.48	**4.47**
$F_{c\infty}$	–	6.12	6.08	6.08	**6.05**
F_{u0}	–	12.43	12.09	12.25	**12.02**
$F_{u\infty}$	–	3.57	3.51	3.52	**3.49**
$F_{\ell 0}$	–	1.42	1.50	1.53	**1.57**
$F_{\ell\infty}$	–	1.30	1.37	1.41	**1.45**
F_w	–	3.75	3.64	3.92	**3.79**
F_m	–	2.02	2.06	2.07	**2.10**
F_v	–	2.37	2.29	2.47	**2.39**
β_{c0}	–	14.87	14.85	14.85	**14.83**
$\beta_{c\infty}$	–	16.82	16.77	16.79	**16.75**
β_{u0}	–	20.56	20.56	20.56	**20.56**
$\beta_{u\infty}$	–	15.79	15.57	15.64	**15.49**
$\beta_{\ell 0}$	–	3.49	4.06	4.33	**4.65**
$\beta_{\ell\infty}$	–	2.50	3.09	3.39	<u>**3.72**</u>
β_w	–	17.25	16.79	17.90	**17.41**
β_m	–	8.01	8.34	8.38	**8.58**
β_v	–	11.34	10.79	12.14	**11.54**

cost increase of 120.77 euros (see the corresponding entry in Table 2). Similarly, an increase in the safety factor lower bound F_{c0} does not change the cost.

Table 3 gives the sensitivities associated with the β-values. It is useful to know how much changes the corresponding β-value when a small change in a single data value is made, for example, the means, standard deviations, etc. In this table the designer can easily analyze how the quality of the material (reduced standard deviations in f_c or f_y) or precission in the applied loads (reduced standard deviations in v_{p_s} or v_{w_p}) influence the safety of the beam.

Table 2: Sensitivities with respect to the data values in the wall illustrative example.

d_{st}	l	r_1	r_2	pl	n_b	e_p	e_u
-483.43	27.53	0.00	0.00	-19.80	0.00	74.17	82.89
γ_c	γ_s	ϕ	E_s	m	w_{dk}	p_{sk}	w_{pk}
4.13	9.13	44.18	0.00	6.17	0.00	0.00	0.00
f_{ck}	f_{sk}	f_{yk}	F_{c0}	$F_{c\infty}$	F_{u0}	$F_{u\infty}$	$F_{\ell 0}$
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$F_{\ell\infty}$	F_w	F_m	F_v	c_y	c_s	c_{st}	c_c
185.65	0.00	0.00	0.00	3276.71	94.02	23.31	0.67

Table 3: Sensitivities $\frac{\partial \beta_i}{\partial x}$, with respect to the data values x, in the wall illustrative example.

Data x	$\frac{\partial \beta_{c0}}{\partial x}$	$\frac{\partial \beta_{c\infty}}{\partial x}$	$\frac{\partial \beta_{u0}}{\partial x}$	$\frac{\partial \beta_{u\infty}}{\partial x}$	$\frac{\partial \beta_{\ell 0}}{\partial x}$	$\frac{\partial \beta_{\ell\infty}}{\partial x}$	$\frac{\partial \beta_{w}}{\partial x}$	$\frac{\partial \beta_{m}}{\partial x}$	$\frac{\partial \beta_{v}}{\partial x}$
l	-0.39	-0.28	0.00	-0.74	-0.66	-0.63	-0.28	-0.77	-0.35
d_{st}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.31	0.00
r_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.31	0.00
pl	0.36	0.25	0.00	0.81	0.58	0.54	0.78	0.73	0.94
n_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
e_p	-1.42	-0.99	0.00	-3.38	-2.24	-2.05	-3.23	-2.92	-3.85
e_u	-1.30	-0.91	0.00	-2.64	-2.03	-1.88	-2.87	-2.64	-3.44
γ_c	-0.05	-0.04	0.00	-0.07	-0.09	-0.09	-0.07	-0.09	-0.09
γ_s	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	-0.01	-0.01
ϕ	0.00	0.58	0.00	-4.44	0.00	-0.36	0.00	0.00	0.00
E_s	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
m	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ_{w_d}	-0.02	-0.01	0.00	-0.03	-0.03	-0.03	-0.03	-0.04	-0.04
μ_{p_s}	-0.55	-0.38	0.00	-1.12	-0.86	-0.80	-1.22	-1.12	-1.46
μ_{w_p}	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
μ_{f_c}	0.17	0.12	0.00	0.12	0.01	0.01	0.00	0.02	0.00
μ_{f_s}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ_{f_y}	0.00	0.00	0.00	0.06	0.05	0.05	0.06	0.06	0.04
v_{w_d}	-0.95	-0.62	0.00	-2.32	-1.18	-0.97	-2.62	-2.03	-3.01
v_{p_s}	-4.13	-2.69	0.00	-10.06	-5.12	-4.20	-11.36	-8.80	-13.04
v_{w_p}	-5.72	-3.72	0.00	-16.29	-7.31	-5.96	-15.72	-12.18	-18.05
v_{f_c}	-240.29	-302.33	-422.67	-23.46	-0.01	-0.02	0.00	-0.27	0.00
v_{f_s}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
v_{f_y}	0.00	0.00	0.00	-199.34	-22.80	-15.92	-292.20	-62.05	-53.85

6 Conclusions

The FPSF methods, when used for designing a parameterized composite beam with given objective function, and used as described in this paper, presents the following advantages:

1. It leads to an *automatic design*, i.e., the values of the design variables are given, not by the engineer, but by the optimization process itself.

2. It leads to an *optimal design*, in the sense of optimizing the given objective function.

3. It leads to a *designer independent design*. It is not dependent of who designs the engineering work.

4. It gives information about actual *safety factors* and *probabilities of failure* for all *failure modes*.

5. It gives information about the *optimal value* of the objective function (cost).

6. It gives the *sensitivities* of the value of the objective function to the data, including costs of materials, fixed geometric variables, safatey factor lower bounds, etc.

7. It gives the *sensitivities* of the β-values to the data, including the failure probability upper bounds, statistical parameters, etc.

8. It works with a *double safety control* (factors of safety and failure probabilities) that leads to a safer and less dependent (more robust) on statistical assumptions, design.

9. It allows an *easy dialog* between classical and probability based designers.

10. It facilitates the understanding of the close *connection between the safety factor and the failure probability* k-dimensional spaces.

7 Acknowledgments

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