An Optimal Engineering Design Method with Failure Rate Constraints and Sensitivity Analysis. Application to Composite Breakwaters

Carmen Castillo[1], Roberto Mínguez[2], Enrique Castillo[3] and Miguel A. Losada[4]

June 14, 2004

Abstract

The paper presents a new method for the design of maritime structures against sea waves that minimizes the initial/construction cost of the structure when the yearly maximum acceptable failure rates for all modes of failure are fixed beforehand. The solution of the resulting optimization problem becomes complex because the evaluation of failure rates involves one optimization problem per failure mode (FORM), so that a decomposition method is used to solve the problem. In addition, a sensitivity analysis is performed, which makes it possible to determine how the cost and yearly failure rates of the optimal solution are affected by small changes in the input data values. The proposed method is illustrated by its application to the design of a composite wall under breaking and non breaking wave conditions. The storms are assumed to be stochastic processes characterized by their maximum significant wave heights, their maximum wave heights and the associated zero-up-crossing mean periods.

Key Words: Cost optimization, Failure probability, Modes of failure, Stochastic process, Reliability analysis, Safety factors.

1 Introduction

The phases that an engineering structure undergoes are: construction, service life and dismantling. In addition, maintenance and repair take place during the service lifetime. During each of these phases, the structure and the environment undergo a continuous sequence of outcomes, the consequences of which have to be considered in the project. The objective of the design is to verify that the structure satisfies the project requirements during these phases in terms of acceptable failure rates and cost (see Losada [1] and ROM [2]).

Since repair depends on the modes of failure and their occurrence frequencies, these must be defined. A mode describes the form or mechanism in which the failure of the structure or one of its elements occurs. Each mode of failure is defined by a corresponding limit state equation as, for example:

$$g_m(x_1, x_2, \dots, x_n) = h_{sm}(x_1, x_2, \dots, x_n) - h_{fm}(x_1, x_2, \dots, x_n); \quad m \in M, \tag{1}$$

[1]Assistant Professor, Grupo de Puertos y Costas, CEAMA, University of Granada, Avda. del Mediterráneo s/n, 18071 Granada, Spain. e-mail: mcastill@ugr.es

[2]Assistant Professor, Department of Applied Mathematics, University of Castilla-La Mancha, 13071 Ciudad Real, Spain. e-mail: Roberto.Minguez@uclm.es & FulBright Scholar, Department of Statistics, Cornell University, 14853-3901 Ithaca (NY), USA. e-mail: rm333@cornell.edu

[3]Full Professor, Department of Applied Mathematics and Computational Sciences, University of Cantabria, Avda. Castros s/n., 39005 Santander, Spain. e-mail: castie@unican.es

[4]Full Professor, Grupo de Puertos y Costas, CEAMA, University of Granada, Avda. del Mediterráneo s/n, 18071 Granada, Spain. e-mail: mlosada@ugr.es

where $(x_1, x_2, \ldots, x_n)$ refer to the values of the variables involved, $g_m(x_1, x_2, \ldots, x_n)$ is the safety margin and $h_{sm}(x_1, x_2, \ldots, x_n)$ and $h_{fm}(x_1, x_2, \ldots, x_n)$ are two opposing magnitudes (such as stabilizing and mobilizing forces, strengths and stresses, etc.) that tend to avoid and produce the associated mode of failure, respectively, and M is the set of all failure modes.

In this paper it is assumed that failure occurs during storms that are assumed to be stochastic processes of random intensity, and that failure occurs when the critical variables (extreme wave heights and periods) satisfy $g_m \leq 0$. Then, the probability of failure mode m in a given period becomes:

$$P_{f_m} = \int\limits_{g_m(x_1,x_2,\ldots,x_n)\leq 0} f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)dx_1dx_2\ldots dx_n, \tag{2}$$

where $f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)$ is the joint probability density function of all variables involved in the problem. With this information, and the consideration of all storms that may occur in a year, the different yearly failure rates for all failure modes can be estimated.

If the design variables lead to admissible failure rates, i.e., below given upper bounds, the design is said to be safe. The main advantage of probabilistic based design is that the reliability of the structure can be evaluated. However, they are very sensitive to tail assumptions (behavior of the random variables for extreme values) (see Galambos [3] and Castillo [4]), and in some cases, as, for example, vertical wall stability, runup, overtopping, geotechnical stability, etc., the dependence structure and the statistical distributions of the variables involved are difficult to define.

Over the last few years design methods have been improved by applying optimization techniques. The main advantage is that these techniques lead to optimal design and automation, i.e., the values of the design variables are provided by the optimization procedure (the optimal values) and not fixed by the engineer. Designer' concerns are only the constraints to be imposed on the problem and the objective function to be optimized.

Some authors consider the construction cost (Castillo et al. [5, 6], Castillo et al. [7, 8, 9]) or the total cost (construction, maintenance and repairs) as the design criteria (Van Dantzig [10], Burchart et al. (1995), Voortman et al. [11], Enevoldsen [12], Enevoldsen and Sorensen [13, 14] and Mínguez et al. [15]). As the main purpose of the different maritime structures is to protect areas from being flood by large waves, and they can be used in very different conditions where the consequences of a partial or complete failure also are very different, the accepted probability of failure varies considerably. However, people should not allow engineers and politicians to make their decision based only on economic criteria. Human life, quality and service reliability, and perhaps other criteria must be considered. In fact, some constraints on the yearly failure probability rate must be imposed. The calculation of which implies solving as many optimization problems as failure modes. Thus, use of optimization programs is not straightforward.

In some cases (see Nielsen and Burcharth [16]) cost evaluations take into account the occurrence of failures, but taking into account the actual sequence of failures is difficult. Large storms produce at most one single failure of each type (mode) or combinations of them, because even though several of its waves (the largest) are able to produce failure, once destroyed, the breakwater cannot be destroyed again before its repair that will take place once the storm has finished. An evaluation of the number of failures must take into consideration that several dangerous sea waves normally occur during the same storm, but produce at most one failure of each type. This implies that the natural event to predict the number of failures is the storm occurrence.

In addition to requiring optimal solutions to problems, some interest is shown by people in knowing how sensitive are the solutions to data values. A sensitivity analysis provides excellent information on the extent to which a small change in the parameters or assumptions (data) modifies the resulting design (geometric dimensions, costs, reliabilities, etc.). This will be useful to: (a) the designer in order to know how sensitive the design is to the assumptions, (b) the construction engineer to know to what extent changes in the unit prices and other data modify the cost and reliabilities, and (c) the code designer to know, for example, how much a lowering of the failure rate bounds increases the cost.

The aims of this paper are: (a) to present a method for evaluating the frequency of failures with their normal sequencing being taken into account, i.e., within storms, (b) to present a design method that minimizes the initial/construction costs subject to some yearly failure rate constraints applicable to composite breakwaters or other type of maritime structures, and (c) to provide tools to perform a sensitivity analysis.

The paper is structured as follows. In Section 2 the probabilistic design is described. In Section 3 the proposed method for optimal design is presented. In Section 4 a technique for performing a sensitivity analysis is explained. Section 5 illustrates the proposed method by an example application dealing with the design of a composite breakwater. Section 6 is devoted to the discussion of the statistical assumptions. Section 7 presents a numerical example. Finally, Section 8 gives some conclusions.

2 The Probabilistic Design Problem

In this section the probabilistic design problem is described.

Safe and failure domains. In the design and reliability analysis of a maritime structure, there are some random variables $(X_1, \ldots, X_n)$ involved. They include geometric variables, material properties, loads, etc. In this paper, without loss of generality, we make no distinction between random and deterministic variables. So, it is assumed that all variables involved are random, and deterministic variables are only particular cases of them. They belong to an n-dimensional space, which, for each mode of failure, can be divided into two domains, the safe and the failure domains:

$$\left.\begin{array}{ll}\text{Safe domain:} & \mathcal{S} \equiv \{(x_1, x_2, \ldots, x_n)\} | g_m(x_1, x_2, \ldots, x_n) > 0\} \\ \text{Failure domain:} & \mathcal{F} \equiv \{(x_1, x_2, \ldots, x_n)\} | g_m(x_1, x_2, \ldots, x_n) \leq 0\}\end{array}\right\}; \quad m \in M \tag{3}$$

where M is the set of all modes of failure m.

It is important to distinguish between design values of the random variables X_i, and actual values x_i $(i = 1, 2, \cdots, n)$. The design values are those values selected by the engineer at the design stage for the geometric variables (dimensions), the material properties (strengths, stiffness, etc.), that do not necessarily correspond with those in the real work. Thus, in this paper the design values are assumed to be the means or the characteristic values (extreme percentiles) of the corresponding random variables, and are denoted $\bar{x}_i$ (mean) and $\tilde{x}_i$ (characteristic), respectively. Some of these design values are chosen by the engineer or given by the design codes, and some (associated with the design variables) are selected by the optimization procedure to be presented. In this paper, the set of variables $(X_1, \ldots, X_n)$ will be partitioned in four sets (for the particular example of the composite breakwater see Appendix A):

1. **Optimized design variables $\boldsymbol{d}$:** Design random variables the mean values of which are to be chosen by the optimization procedure to optimize the objective function (minimize the total expected cost). Normally, they describe the dimensions of the work being designed, such as width, thickness, height, cross sections, etc., but can include material properties, etc.

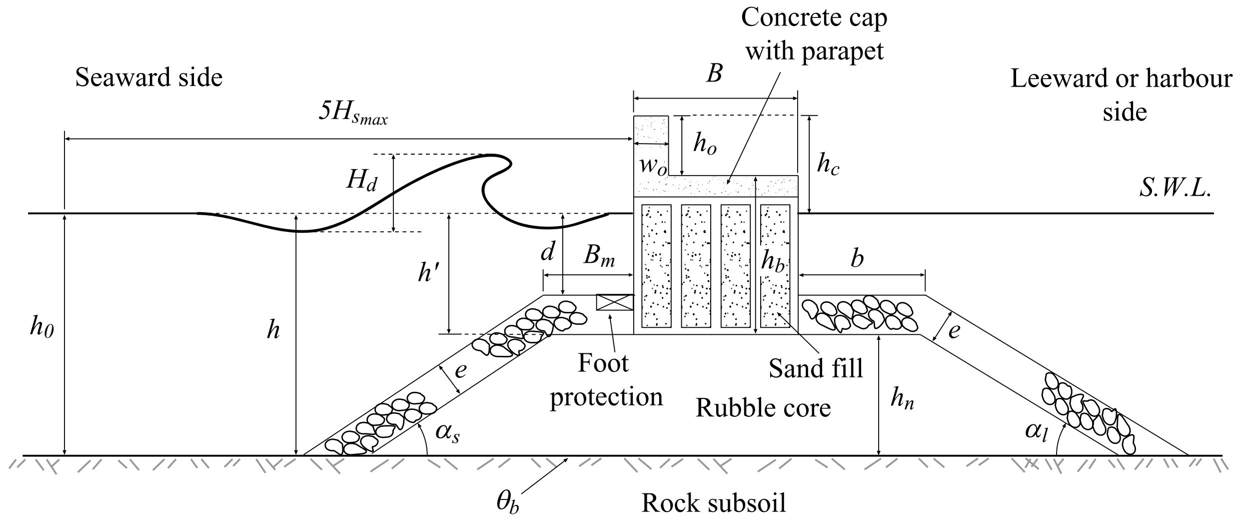

Figure 1: Composite breakwater showing the geometric design variables.

2. **Non-optimized design variables $\boldsymbol{\eta}$:** Set of variables the mean or characteristic values of which are fixed by the engineer or the code guidelines as input data to the optimization program. Some examples are costs, material properties (unit weights, strength, Young modulus, etc.), and other geometric dimensions of the work being designed (parapet breakwater width, etc.) that are fixed.

3. **Random model parameters $\boldsymbol{\kappa}$:** Set of parameters used in the probabilistic design, defining the random spatial and temporal variability and dependence structure of the variables involved (standard deviations, variation coefficients, correlations, etc.).

4. **Dependent or non-basic variables $\boldsymbol{\psi}$:** Dependent variables which can be written in terms of the basic variables $\boldsymbol{d}$ and $\boldsymbol{\eta}$ to facilitate the calculations and the statement of the problem constraints.

The corresponding means of $\boldsymbol{d}$ will be denoted $\bar{\boldsymbol{d}}$, and the mean or the characteristic values of $\boldsymbol{\eta}$ is denoted $\tilde{\boldsymbol{\eta}}$.

The cost optimization problem to be stated in Section 3 will make use of these sets of variables.

Given a set of values of the design variables $\bar{\boldsymbol{d}}$, the probability of failure p_{st}^m under mode m during a random storm can be calculated using the joint probability density function $f(\boldsymbol{x}) = f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n; \boldsymbol{\theta})$ of all variables involved, where $\boldsymbol{\theta}$ is a parametric vector, by means of the integral:

$$p_{st}^m(\boldsymbol{\theta}) = \int\limits_{g_m(x_1,x_2,\ldots,x_n)\leq 0} f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n; \boldsymbol{\theta}) dx_1 dx_2 \ldots dx_n. \tag{4}$$

In this paper we assume that the parametric vector $\boldsymbol{\theta} = (\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ contains the means $\bar{\boldsymbol{d}}$, the means or the characteristic values $\tilde{\boldsymbol{\eta}}$, and some other extra vector of random model parameters $\boldsymbol{\kappa}$.

Unfortunately, calculation of $p_{st}^m(\boldsymbol{\theta})$ is difficult. So, to eliminate the need for complex numerical integrations, the "First Order Reliability Methods" (FORM) transform the initial set of variables into an independent multinormal set and use a linear approximation. For a complete description of some of these methods and some illustrative examples see Hasofer and Lind [17], Madsen, Krenk and Lind [18], Ditlevsen and Madsen [19], or Melchers [20], and for maritime engineering see Burcharth [21, 22, 23], Burcharth and Sorensen [24], Goda [25] and Goda and Takagi [26].

In this paper we assume that the reader is familiar with the FORM for evaluating the probability of failure, more precisely, $p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ for $m = 1, 2, \ldots, M$ is obtained using:

$$p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \underset{\boldsymbol{d}, \boldsymbol{\eta}}{\text{Maximum}} \quad \Phi(-\beta_m) = \Phi(-\sqrt{\boldsymbol{z}^T\boldsymbol{z}}) \ , \tag{5}$$

i.e., maximizing with respect to $\boldsymbol{d}, \boldsymbol{\eta}$, subject to

$$\boldsymbol{z} = \mathbf{G}(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}, \boldsymbol{\theta}) \tag{6}$$

$$\boldsymbol{q}(\boldsymbol{d}, \boldsymbol{\eta}) = \boldsymbol{\psi} \tag{7}$$

$$g_m(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}) = 0, \tag{8}$$

where β_m is the reliability index for failure mode m, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable, $G(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}, \boldsymbol{\theta})$ is the transformation leading to the standard unit normal $\boldsymbol{z}$ variables used in FORM, $\boldsymbol{q}(\boldsymbol{d}, \boldsymbol{\eta}) = \boldsymbol{\psi}$ are the equations that allow obtaining the values of the intermediate variables $\boldsymbol{\psi}$, and $g_m(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}) = 0$ is the boundary of the failure region for failure mode m.

Note that we do not minimize β_m in (5) as usual, but maximize the probability of failure $\Phi\left(-\sqrt{\mathbf{z}^T\mathbf{z}}\right)$. However, since the functions $\Phi(\cdot)$ and square root are increasing, both approaches are equivalent. The second has been chosen because we later look for the probability of failure sensitivities with respect to the data, i.e., the rate of change of $\Phi\left(-\beta_m\right)$ with respect to the data values.

3 Proposed Method for Optimal Design

To design the maritime structure we propose to minimize the initial/construction cost subject to failure rate constraints. Since the latter involves random occurrences, some model assumptions are necessary. Note that contrary to the material in Section 2, that is well known, some of the formulas and the model to be presented in Sections 3 and 4 are original.

3.1 Model assumptions

Before describing the model assumptions, it is worth mentioning that the aim of this paper is to introduce a new approach of breakwater design based on minimizing initial/construction cost subject to yearly failure rates bounds for all failure modes, and to present a technique for sensitivity analysis. Thus, the breakwater example to be discussed below is simply an illustrative example, and it must be considered as such because the analysis can not be considered exhaustive, since several failure modes were not implemented (settlement, scour, deterioration and corrosion of reinforcement due to chloride ingress through the concrete or concrete cracks, etc.) and some hydraulic responses were not analyzed (wave transmission, wave reflection).

Our model is based on the following assumptions:

1. The storms are assumed to be stochastic processes, i.e., to occur at random times with yearly rate r_{st} (mean number of storms per year). Note that no assumption is needed about the dependence or independence of storms or the distribution of occurrence times, because only the yearly failure rate is looked for.

2. Long-term statistics deal with the distribution of the storms which are characterized by a set of three variables that represent the maximum significant wave height $H_{s_{max}}$ of all its sea states, its maximum wave height H_{max}, and the associated wave period $T_{z_{max}}$ (that occurring with H_{max}). It is assumed that they are dependent random variables whose probability distribution and dependence structure must be derived from real data. Once a storm has occurred, its intensity and characteristics can be derived from this joint distribution, i.e., a set of values $\{H_{s_{max}}, H_{max}, T_{z_{max}}\}$ can be drawn at random from a population with the corresponding distribution. For the sake of simplicity, we assume that these variables provide enough information to verify the breakwater failure modes.

3. Failures occur during storms and the probability of failure in mode m in a random storm is $p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$, which has been considered to be a function of the design variables and parameters $(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$, which include the geometric dimensions of the breakwater and the parameters defining the probability distribution of all variables involved.

4. One storm can cause at most only one failure of each type (mode), because in the case of occurrence of several sea waves in one storm all able to produce failure, only the first failure of each mode must be considered, because repair is not possible during storms. This implies that failure accumulation is not included.

5. A failure mode does not induce any other failure modes. This means that the structure is assumed not to suffer a progressive collapse. However, different failure modes can occur simultaneously, and they are not statistically independent because they have common inducing agents.

   Interaction between failure modes is an important problem. However, we have to bear in mind that nowadays there is not enough knowledge on such interaction for it to be included in models; we are still trying to understand and to evaluate how individual modes of failure start and progress. Thus, to complicate the presentation of a new optimization procedure with additional heuristic approaches is in the authors opinion not the best decision for this paper, though, for example, a model for interaction between the toe berm and the main armour for rubble mound breakwaters is presented in Christiani [27].

6. The probability of failure in mode m, $p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$, is a Benoulli random variable, and the mean number of storms per year is r_{st}. Thus, the mean number of failures per year is equal to the mean of the Binomial random variable $B(r_{st}, p_{st}^m)$,

$$r_{st} p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}). \tag{9}$$

7. The proposed approach is based on guaranteeing bounded yearly failure rates of all failure modes. However, for the global failure rate, one can consider the well known bounds:

$$\text{Lower bound: } P_f = \max_m P_{f_m}; \ \text{ Upper bound: } P_f = 1 - \prod_{m=1}^{M} (1 - P_{f_m})$$

   where P_f is the global probability of failure (upper bound failure rates could be included in the proposed method without additional effort).

3.2 Initial/construction cost function

In this paper the criteria for design is based on minimizing the initial/construction cost per running meter of vertical breakwater subject to bounded yearly failure rates. The objective function is a function of the volumes of sand, concrete and rubble stone in the caisson section. In this paper we do not consider cost of damage due to serviceability service states and to ultimate limit states (see Sorensen et al. [28], Voortman et al. [11]) but these cost are bounded by the yearly failure rate bounds. Thus, the initial/construction cost ($C_0(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})$), is given by

$$C_0(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \quad = \quad C_c V_c + C_{a\ell} V_{a\ell} + C_{co} V_{co} \tag{10}$$

where $\bar{\boldsymbol{d}}$ and $\tilde{\boldsymbol{\eta}}$ are the design variables at their means and characteristic values, respectively, $V_c, V_{a\ell}$ and V_{co} are the sand filled caissons, armor layer, and core volumes, respectively, and $C_c, C_{a\ell}$ and C_{co} are the respective construction costs per unit volume. The details of the derivation of the cost function are given in Appendix B.

3.3 Evaluation of the failure mode probabilities in a random storm

In this paper we evaluate the failure mode probabilities in a random storm using first order reliability methods (FORM). More precisely, $p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ for $m = 1, 2, \dots, M$ is obtained using:

$$p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \underset{\boldsymbol{d}, \boldsymbol{\eta}}{\text{Maximum}} \quad \Phi(-\beta_m) = \Phi(-\sqrt{\boldsymbol{z}^T \boldsymbol{z}}) \tag{11}$$

that is maximizing with respect to $\boldsymbol{d}, \boldsymbol{\eta}$, subject to

$$\boldsymbol{z} \quad = \quad \mathbf{G}(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\theta}) \tag{12}$$

$$\boldsymbol{q}(\boldsymbol{d}, \boldsymbol{\eta}) \quad = \quad \boldsymbol{\psi} \tag{13}$$

$$g_m(\boldsymbol{d}, \boldsymbol{\eta}) \quad = \quad 0, \tag{14}$$

where β_m is the reliability index for failure mode m, $G(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\theta})$ is the transformation leading to the standard unit normal distribution, and $g_m(\boldsymbol{d}, \boldsymbol{\eta}) = 0$ is the boundary of the failure region for failure mode m.

Note that we do not minimize β_m in (11) as usual, but maximize the probability of failure $\Phi\left(-\sqrt{\mathbf{z}^T \mathbf{z}}\right)$. However, since the functions $\Phi(\cdot)$ and square root are increasing, both approaches are equivalent. The second has been chosen because we later look for the probability of failure sensitivities with respect to the data, i.e., the rate of change of $\Phi\left(-\beta_m\right)$ with respect to the data values.

Once the probabilities for all failure rates have been calculated it possible to obtain the yearly failure rates for all modes. Thus, once the failure rates bounds are decided their incorporation into the optimization procedures as additional constraints can be done as follows:

$$\mathbf{r}_m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = r_{st} p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) \leq R_m^0, \tag{15}$$

where $R_i^0; i = 1, 2, \dots, M$ are the corresponding failure rates upper bounds for the M different failure modes, which should be fixed by the codes.

Everything is now ready to state the design problem as an optimization problem as follows.

3.4 Design as an optimization problem

In this paper the design of a maritime structure is equivalent to solve the following optimization problem:

$$\underset{\bar{\boldsymbol{d}}}{\text{Minimize}} \quad C_0(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \ , \tag{16}$$

i.e., minimize with respect to $\bar{\boldsymbol{d}}$, subject to the yearly failure rate, the equations that allow obtaining the intermediate variables, and geometric constraints:

$$\begin{aligned} r_m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = r_{st} p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) &\leq R_m^0; \quad m = 1, \cdots, M & (17)\\ \boldsymbol{q}(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) &= \boldsymbol{\psi} & (18)\\ \boldsymbol{h}(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\psi}) &\leq \mathbf{0}. & (19) \end{aligned}$$

where $p_{st}^m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ is given by solving the problem (11)-(14). The constraints (17) are called complicating constraints, because they involve inner optimization problems.

3.5 Solving the cost optimization problem using decomposition techniques

The problem described in Eqs. (16)-(19) presents some difficulties because constraints (17) require the knowledge of $\mathbf{r}_m(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$, the calculation of which implies solving several optimization problems (11)-(14) (one per failure mode).

This type of problem can be solved using decomposition techniques (see Benders [29] and Geoffrion [30]) that were applied to reliability optimization problems by Mínguez [31], and Mínguez et al. [32, 15]. The price that has to be paid for such a simplification is iteration. That is, instead of solving the original problem at once, two simpler problems are solved iteratively: a simple called master problem which is a problem similar to the original one but replacing the probabilities of failure for each failure mode by linear approximations, and a subproblem or subproblems (one for each failure mode) where the linear approximations of the probabilities of failure are updated for the new design values obtained form the master problem. For a detailed analysis of decomposition techniques see Conejo et al. [**?**]. The use of this method together with FORM for the reliability evaluations holds is not a time consuming method and the values of the failure rate functions are stable for any given point.

The following iterative scheme, which solves two optimization problems (the master problem and the subproblems) can be applied to solve the problem (16)-(19):

- **Step 0: Initialization.** Initialize the iteration counter $\nu = 1$, select some initial values for the design variables $\bar{\boldsymbol{d}} = \bar{\boldsymbol{d}}_1$ and evaluate the initial/construction cost $C_0^{(1)} = C_0(\bar{\boldsymbol{d}}_1, \tilde{\boldsymbol{\eta}})$. To improve convergence it is convenient that initial design hold the failure rate requirements (17).

- **Step 1: Subproblem solution.** Solve the subproblems, i.e., the problems (11)-(14) modified to

$$r_m(\bar{\boldsymbol{d}}_\nu, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \underset{\boldsymbol{d}, \boldsymbol{\eta}, \bar{\boldsymbol{d}}}{\text{Maximum}} \quad r_{st}\Phi(-\beta_m) = r_{st}\Phi(-\sqrt{\boldsymbol{z}^T\boldsymbol{z}}) \tag{20}$$

subject to

$$\begin{aligned} \boldsymbol{z} &= \mathbf{G}(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}, \bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) & (21)\\ g_m(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}) &= 0, & (22) \end{aligned}$$

$$\boldsymbol{q}(\boldsymbol{d}, \boldsymbol{\eta}) = \boldsymbol{\psi} \quad (23)$$

$$\bar{\boldsymbol{d}} = \bar{\boldsymbol{d}}_\nu \; : \; \boldsymbol{\mu}_{m\nu} \quad (24)$$

where the corresponding dual variables have been denoted $\boldsymbol{\mu}_{m\nu}$.

- **Step 2: Master problem solution for iteration ν.** The master problem which consist in replacing the yearly failure rates per a linear approximations in problem (16)-(19) is solved:

$$\underset{\bar{\boldsymbol{d}}}{\text{Minimize}} \quad C_0(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \; , \quad (25)$$

subject to

$$r_m^*(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) \leq R_m^0; \quad m = 1, \cdots, M \quad (26)$$

$$\boldsymbol{q}(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \boldsymbol{\psi} \quad (27)$$

$$\boldsymbol{h}(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\psi}) \leq \boldsymbol{0} \quad (28)$$

$$r_m^*(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = r_m(\bar{\boldsymbol{d}}_{\nu-1}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) + \boldsymbol{\mu}_{m\nu-1}^T(\bar{\boldsymbol{d}} - \bar{\boldsymbol{d}}_{\nu-1}); \quad m = 1, \cdots, M, \quad (29)$$

obtaining $\bar{\boldsymbol{d}}_\nu$ and $C_0^{(\nu)} = C_0(\bar{\boldsymbol{d}}_\nu, \tilde{\boldsymbol{\eta}})$. Note that $r_m^*(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ is a linear approximation of the yearly failure rate.

- **Step 3: Convergence checking.** If $\left| \dfrac{C_0^{(\nu)} - C_0^{(\nu-1)}}{C_0^{(\nu)}} \right|$ is lower than the tolerance, the procedure stops, otherwise, go to **Step 2**.

The process of solving iteratively these schemed is repeated until convergence. Observe also that approximative hyperplanes (29) are constructed using the partial derivatives of the yearly failure rates ($\boldsymbol{\mu}_{mk}$) with respect the design variables ($\bar{\boldsymbol{d}}$).

4 Sensitivity Analysis

The problem of sensitivity analysis in reliability based optimization has been discussed by several authors, see, for example, Enevoldsen [33], or Sorensen and Enevoldsen [34]. In this section we show how the duality methods can be applied to sensitivity analysis in a straightforward manner. We emphasize here that the method to be presented in this section is of general validity.

In the problem (25)-(29) it is very easy to obtain the sensitivities of the optimal initial/construction cost (the objective function) with respect to the failure rate bounds R_m^0 because they appear on the right hand side of constraint (26). When this happen, this sensitivity is simply the value of the dual variable associated with that constraint, that practically all software optimization packages give by free because it is very easy to calculate once the optimal solution has been found.

The problem arises when the data or parameters with respect to which we want to calculate the sensitivities do not appear on the right hand side of a constraint.

The way of solving this problem consists of generating artificial (redundant) constraints that satisfy such a condition. One way of generating these constraints consists of transforming all the parameters or data with respect to which we desire the sensitivities, into artificial variables and adding the constraints that lock the variables to their actual values. To illustrate, we apply this technique to the optimization problems (20)-(24) and (25)-(29) at the optimal solution $\bar{\boldsymbol{d}}^*$.

The problem (20)-(24) is obviously equivalent to the problem

$$\underset{\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{d}^*, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*}{\text{Maximum}} \quad r_{st}\Phi(-\beta_m) = r_{st}\Phi(-\sqrt{\boldsymbol{z}^T\boldsymbol{z}}) \tag{30}$$

subject to

$$\begin{aligned}
\boldsymbol{z} &= \mathbf{G}(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*, \boldsymbol{d}^*) && (31)\\
\boldsymbol{q}(\boldsymbol{d}, \boldsymbol{\eta}) &= \boldsymbol{\psi} && (32)\\
g_m(\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\psi}) &= 0, && (33)\\
\boldsymbol{d}^* &= \bar{\boldsymbol{d}}^* : \ \boldsymbol{\mu}_m && (34)\\
\boldsymbol{\eta}^* &= \tilde{\boldsymbol{\eta}} : \ \boldsymbol{\delta}_m && (35)\\
\boldsymbol{\kappa}^* &= \boldsymbol{\kappa} : \ \boldsymbol{\xi}_m && (36)
\end{aligned}$$

where $\boldsymbol{d}^*$, $\boldsymbol{\eta}^*$ and $\boldsymbol{\kappa}^*$ are the artificial variables.

The basic idea is simple. Assume that we wish to know the sensitivity of the objective function to changes in some data values $\bar{\boldsymbol{d}}^*$, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$. Converting the data into artificial variables, $\boldsymbol{d}^*$, $\boldsymbol{\eta}^*$ and $\boldsymbol{\kappa}^*$, and locking them, by means of constraints (34)-(36), to their actual values $\bar{\boldsymbol{d}}$, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$, we obtain a problem that is equivalent to the initial optimization problem but has a constraint such that the values of the dual variables associated with them give the desired sensitivities. More precisely, the values of the dual variables $\boldsymbol{\mu}_m$, $\boldsymbol{\delta}_m$ and $\boldsymbol{\xi}_m$ associated with constraints (34)-(36) give the sensitivities of the probability of failure to $\bar{\boldsymbol{d}}^*$, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$, respectively.

These sensitivities allow determining how the reliability of the breakwater changes when its design values and the statistical parameters of the random variables involved are modified.

Similarly, the problem (25)-(29) is obviously equivalent to the problem

$$\underset{\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*}{\text{Minimize}} \quad C_0(\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*) \ , \tag{37}$$

i.e., minimize with respect to $\bar{\boldsymbol{d}}$, subject to the yearly failure rate and geometric constraints:

$$\begin{aligned}
r^*_m(\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*) &\le R^0_m; \quad m = 1, \cdots, M && (38)\\
\boldsymbol{q}(\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*) &= \boldsymbol{\psi} && (39)\\
\boldsymbol{h}(\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*, \boldsymbol{\psi}) &\le \mathbf{0} && (40)\\
r^*_m(\bar{\boldsymbol{d}}, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*) &= r_m(\bar{\boldsymbol{d}}^*, \boldsymbol{\eta}^*, \boldsymbol{\kappa}^*) + \boldsymbol{\mu}^T_{m\nu-1}(\bar{\boldsymbol{d}} - \bar{\boldsymbol{d}}^*) && (41)\\
&\quad + \boldsymbol{\delta}^T_{m\nu-1}(\boldsymbol{\eta}^* - \tilde{\boldsymbol{\eta}}) + \boldsymbol{\xi}^T_{m\nu-1}(\boldsymbol{\kappa}^* - \boldsymbol{\kappa}); \quad m = 1, \cdots, M, && (42)\\
\boldsymbol{\eta}^* &= \tilde{\boldsymbol{\eta}} && (43)\\
\boldsymbol{\kappa}^* &= \boldsymbol{\kappa} && (44)
\end{aligned}$$

where now $\boldsymbol{\eta}^*$ and $\boldsymbol{\kappa}^*$ are the artificial variables.

The values of the dual variables associated with constraints (38), (43) and (44) give the sensitivities of the initial/construction cost to R^0_m, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$, respectively.

These sensitivities allow determining how the initial/construction cost of the breakwater changes when the reliability bounds, its geometric dimensions and the statistical parameters of the random variables are modified.

Remark 1 *Note that problems (30)-(36) and (37)-(44) need to be solved only once, i.e., after solving problem (16) and (19). Because the starting point is already the optimal solution, convergence is ensured at the first iteration.*

5 Optimized Design of a Composite Breakwater

The probability based design of composite breakwaters has been studied by Christiani et al. [35], Burcharth and Sorensen [24], Sorensen and Burcharth [36], as well as in the European project PROVERBS (see Oumeraci et al. [37]) and the PIANC Working Group 28 on Breakwaters with Vertical and Inclined Concrete Walls [38].

In this section the proposed procedure is applied to the design of a composite breakwater. The main section of the breakwater is shown in Figure 1 where the main parameters are shown. Notice that these parameters define geometrically the different elements of the cross section and must be defined in the construction drawings. Our goal is an optimal design based on minimizing the construction and repair costs per running meter of the composite breakwater.

5.1 Modes of failure

In this study a total of 8 modes of failure has been considered: sliding failure (s), turning failure (t), 4 foundation failures (b, c, d, sea), overtopping failure (o), and seaside berm instability failure (a) as it is shown in Figure 2. But other failure modes, such as settlement, scour, deterioration and corrosion of reinforcement due to chloride ingress through the concrete or concrete cracks, wave transmission, wave reflection, etc. could have been considered.

All modes of failure are ascribed to ultimate limit states but the consequences of failure under each mode are considered different. Like other disciplines of civil engineering the occurrence of the failure does not necessarily mean that the structure will collapse but that its resistance is seriously diminished and its functionality seriously affected. Some of those modes are correlated, because they have common agents, or because one mode can induce the occurrence of others. Only the correlation due to common agents is considered in this paper.

The external wave forces on the upright section are the most important considerations in the design of vertical breakwaters, including both pulsating and impact wave loads. The well known Goda pressure formulas (see Goda [25]) for the evaluation of the forces acting on the breakwater (see Figure 2) have been used in this paper. But as the impulsive pressure coefficient used in Goda's formula does not accurately estimate the effective pressure due to impulsive pressure under all conditions the new impulsive pressure coefficient proposed by Takahashi et al. [39] is used. The maximum wave height (H_{max}) is adjusted in the surf zone due to random wave breaking as described by Goda [25]

$$\frac{H_{max}}{L_0} \le A\left\{1 - \exp\left(-1.5\frac{\pi h_0}{L_0}(1 + 15\tan^{4/3}\theta_b)\right)\right\} \tag{45}$$

where h_0 is the water height in the distance of five times the maximum significant wave height $H_{s_{max}}$ toward the offshore of the breakwater, L_0 is the deep water wave length, θ_b is the mean angle of the sea bottom and the coefficient A takes different values depending of the kind of waves, for example, it takes the value 0.17 for regular waves. Its upper and lower limits are 0.18 and 0.12, respectively.

Thus the design wave height H_d is

$$H_d = \min(H_{max}, H_{break}). \tag{46}$$

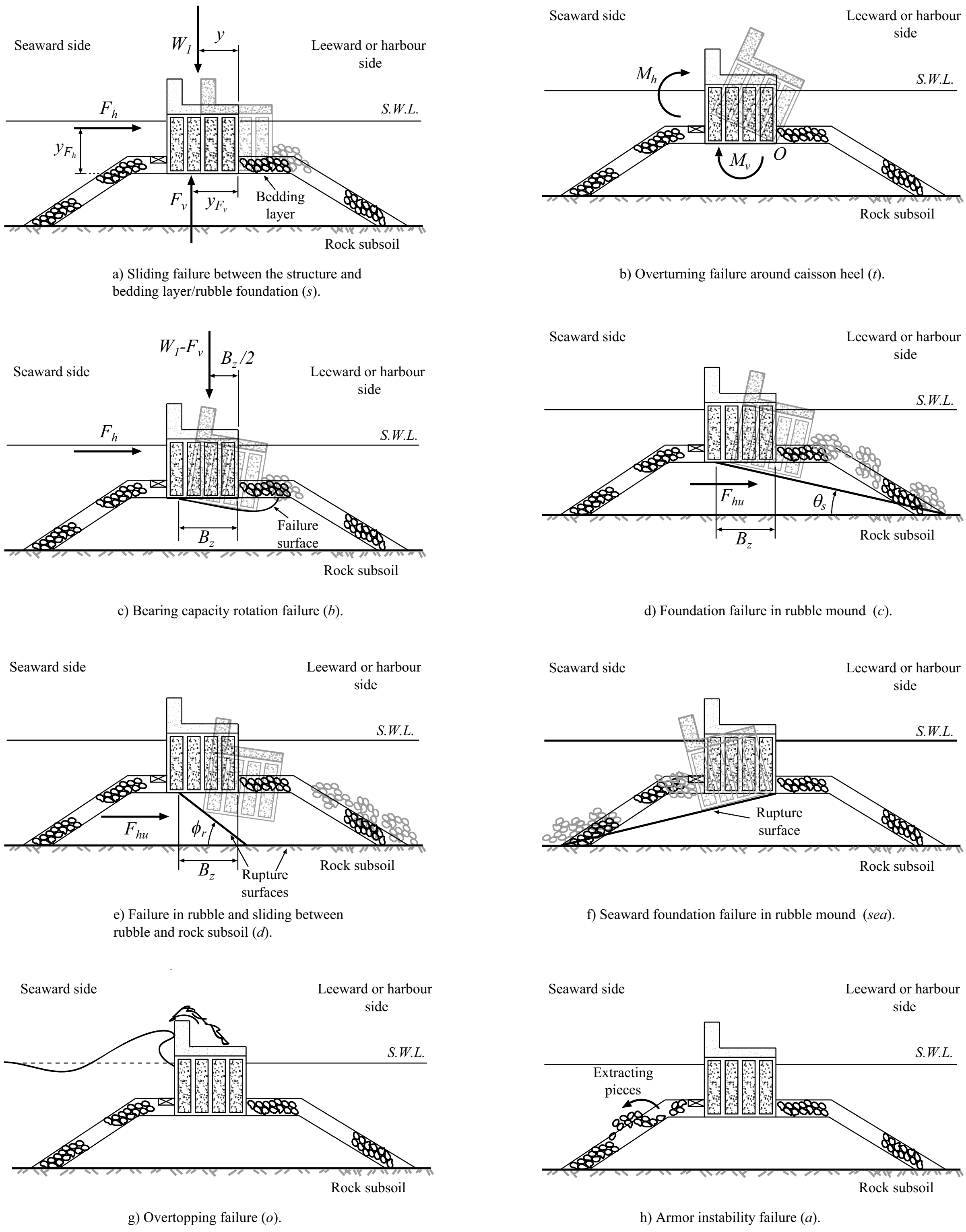

Figure 2: The eight composite failure modes considered in the breakwater example.

Sliding failure. This failure occurs when the breakwater caisson suffers an horizontal displacement, it can occur as a slip either at the interface between the caisson concrete base and the rubble material, or entirely in the rubble material. The safety against sliding failure can be verified by the following limit state equation (see Figure 2(a))

$$g_s = \min(\mu_c, \tan(\phi_r))(W_1 - F_v) - F_h, \tag{47}$$

where μ_c is the friction coefficient, ϕ_r is the angle of internal friction of rubble, F_h and F_v are the total vertical and horizontal forces due to wave pressure, and W_1 is the actual caisson weight reduced for buoyancy, which are given by:

$$\begin{aligned} F_h &= h_c(p_1+p_4)/2 + h'(p_1+p_3)/2 && (48)\\ F_v &= \frac{1}{2}p_u B && (49)\\ W_1 &= V_c\gamma_c - h'B\gamma_w && (50)\\ V_c &= Bh_b + w_o h_o && (51) \end{aligned}$$

where h_c is the freeboard, p_1, p_3 and p_4 are the Goda's pressures at the water level, caisson's bottom and freeboard, respectively, p_u is the uplift pressure, B the caisson width, V_c is the total caisson volume, γ_c is the average unit weight of caisson, h' is submerged height of the caisson, γ_w is the water unit weight, h_b is caisson height, and h_o and w_o are the parapet breakwater height and width, respectively.

Overturning failure. This failure occurs when the breakwater structure rotates with respect to point O (see Figure 2(b)) because of water pressure forces. Note that this failure is relevant only in the cases of monolithic structures placed on very strong foundation soils or rock. Usually this mode is dominated by the bearing capacity failure in the rubble mound. The safety against turning failure can be verified by the following limit state equation

$$g_t = W_1 y - M_v - M_h, \tag{52}$$

where y is the W_1 offset with respect to point O, and M_v and M_h are the moments with respect to point O of the vertical and horizontal water pressure forces, which are given by

$$M_v = \frac{2}{3}F_v B = \frac{1}{3}p_u B^2 \tag{53}$$

and

$$M_h = \frac{1}{6}(2p_1+p_3)h'^2 + \frac{1}{2}(p_1+p_4)h'h_c + \frac{1}{6}(p_1+2p_4)(h_c)^2. \tag{54}$$

Foundation failure. The following geotechnical failure functions for a feasibility level of sophistication proposed by Oumeraci et al. [37] considering that the subsoil material is rock are used in this paper:

1. Rotation failure (b).
2. Rupture surface through rubble only (c).
3. Rupture surface through rubble and along top of subsoil (d).

4. Additionally, we have also considered the seaward rupture surface through rubble only (sea).

The set of failure modes consists of a limited number of failure surfaces with a known a-priori geometry (see Figures 2 (c)-(f)). Alternatively, more sophisticated equations based on the upper bound theory can be used (see Sorensen and Burchart [36], Oumeraci et al. [37]).

It is often very practical to consider the equilibrium of the wall separately from the equilibrium of the soil, thus the integrated effective stresses acting on the skeleton of rubble foundation are obtained as resultant from the other forces acting on the wall. The distance of the vertical force $W_1 - F_v$ component to the harbour side edge B_z is:

$$B_z = 2\frac{W_1 y - F_h y_{F_h} - F_v y_{F_v}}{W_1 - F_v}, \tag{55}$$

where y_{F_h} and y_{F_v} are the lever arms of F_h and F_v, respectively.

The resulting horizontal seepage force in the rubble mound (F_{hu}) can be obtained under the assumptions of triangular pressure distribution in the horizontal direction and hydrostatic pressure in the vertical direction as:

$$F_{hu} = \frac{B_z^2 \tan\theta_s}{2B} p_u \ \text{ or } \ F_{hu} = \frac{h_n(2B_z - h_n/\tan\theta_s)}{2B} p_u, \tag{56}$$

which are valid if $B_z \leq h_n/\tan\theta_s$ or $B_z > h_n/\tan\theta_s$, respectively, where h_n is the core height, and θ_s is the angle between the bottom of the wall and the rupture surface (see Figure 2 (d)), that can be obtained as

$$\theta_s = \arctan\frac{h_n}{B_z + b + (h_n + e)\cot\alpha_\ell}, \tag{57}$$

where b is the leeward berm width, e is the armor layer thickness and α_ℓ is leeward slope angle.

Then, the safety against rotation failure can be verified by the following limit state equation (see Figure 2 (c))

$$g_b = B_z^2(\gamma_s - \gamma_w)\tan\phi_r\left(\tan^2(\pi/4 + \phi_r/2)\exp(\pi\tan\phi_r) - 1\right) - (W_1 - F_v)\left(\frac{1}{1 - F_h/(W_1 - F_v)}\right)^3, \tag{58}$$

where γ_s is the rubble mound unit weight.

The safety against rupture surface through rubble only failure can be verified by the following limit state equation (see Figure 2 (d))

$$g_c = W_1 - F_v + (\gamma_s - \gamma_w)[(B_z + b + e\cot\alpha_\ell)h_n/2 + (b + e\cot\alpha_\ell/2)e] - (F_h + F_{hu})\cot(\phi_r - \theta_s). \tag{59}$$

The safety against rupture surface through rubble and along top of subsoil failure can be verified by the following limit state equation (see Figure 2 (e))

$$\begin{aligned} g_d \ = \ & (W_1 - F_v + (\gamma_s - \gamma_w)[(2(B_z + b + e\cot\alpha_\ell) + h_n(\cot\alpha_\ell - \cot\alpha_{\phi_r}))h_n/2 + (b + e\cot\alpha_\ell/2)e])\,\mu_s \\ & -(F_h + F_{hu}), \end{aligned} \tag{60}$$

where μ_s is the friction coefficient between the rubble bedding layer and the rock subsoil. Note that in this case the angle between the bottom of the wall and the rupture surface is ϕ_r (see Figure 2 (e)),

In addition, for avoiding the seaward failure in calm sea wave conditions, the rupture surface through rubble only is considered using the following limit state (see Figure 2 (f)) equation

$$g_{sea} = \phi_r - \arctan\left(\frac{h_n}{B + B_m + (e + h_n)\cot\alpha_s}\right), \tag{61}$$

where B_m is the seaward berm width and α_s is the seaward slope angle. Note that no wave forces are considered in this failure mode, so the yearly probability treatment will be different than the other failure modes (it does not depend on r_{st}).

Overtopping failure. For a composite breakwater of seaboard slope $\tan \alpha_s$ and freeboard h_c, (see Figure 2 (g)), and a sea state defined by a significant maximum wave height $H_{s_{max}}$, the mean overtopping volume q per unit of breakwater length is given, for a caisson breakwater, by the exponential relation (see Franco and Franco [40])

$$q = a \exp(-b_o h_c / H_{s_{max}}) \sqrt{g H_{s_{max}}^3}, \tag{62}$$

where $q/\sqrt{g H_{s_{max}}^3}$ is the dimensionless discharge, $h_c/H_{s_{max}}$ is the relative freeboard, and a and b_o are coefficients that depend on the structure shape and on the water surface behavior at the seaward face.

The definition of tolerable limits for overtopping is still an open question, given the high irregularity of the phenomenon and the difficulty of measuring it and its consequences. Different levels from functional safety (serviceability limit states) to structural safety (ultimate limit states) mainly in cast in situ concrete superstructures could be considered (see Goda [25] and Franco et al. [41]). In this paper we have just considered the structural damage.

The safety against overtopping failure can be verified from the following equation:

$$g_o = q_0 - q, \tag{63}$$

where q_0 is the maximum allowable mean overtopping discharge for structural damage.

Berm instability failure. It is customary in caisson breakwater construction to provide a few rows of foot-protection concrete blocks at the front and rear of the upright section. It usually consists of rectangular blocks weighting form 100 to 400 kN depending on the design wave height. This protection is indispensable especially against oblique wave attack. The remainder of the berm and slope of the rubble mound foundation must be protected with armor units of sufficient weight to withstand the wave action. In this paper we take into account only the stability of the berm and the slope, so berm instability failure refers to the removal of pieces from the berm and slope as it is shown in Figure 2 (h).

Based on experiments, Losada [1] and following Tanimoto, Yagyu and Goda [42], proposed the following limit state equation to evaluate the dimensionless quantity $\dfrac{W}{\gamma_w H_d^3}$:

$$\frac{W}{\gamma_w H_d^3} = R\Phi_e, \tag{64}$$

where Φ_e is the berm stability function, R is an dimensionless constant, which depends on γ_s (for rubble armor units) and γ_w, and W is the individual armor block weight of the berm, that are given by

$$W = \gamma_s \ell_e^3 \tag{65}$$

$$R = \frac{\gamma_s/\gamma_w}{\left(\dfrac{\gamma_s}{\gamma_w} - 1\right)^3} \tag{66}$$

$$\Phi_e = \min\left\{0.3, \left[4.2\frac{(1-c)d}{c^{1/3}H_d} + 3.24\exp\left(-2.7\frac{d(1-c)^2}{H_d c^{1/3}}\right)\right]^{-3}\right\} \quad (67)$$

$$c = \frac{4\pi d}{L\sinh\left(\frac{4\pi d}{L}\right)}\sin^2\left(\frac{2\pi B_m}{L}\right), \quad (68)$$

where l_e is the equivalent cubic block side, d is the berm depth in front of the caisson, c is an auxiliary variable, and B_m is the seaward width. Under such a conditions, the occurrence of failure can be determined from the following equation:

$$g_a = W - \gamma_w R \Phi_e H_d^3. \quad (69)$$

Remark 2 *Note that once the optimal solution has been obtained, the limit state equations (47), (52), (58), (59), (60), (61), (63) and (69) allow one to determine global and sets of partial safety factors which are equivalent to the reliability constraints in the sense of leading to the same optimal solution (see PIANC, Working Group 28 [38] and Burcharth [23]).*

In fact, the proposed method can be extended to include global and partial safety factors. The authors are working in this line that is the aim of another paper.

5.2 Practical design criteria

In maritime works there are some rules of good practice that should be observed. Some of them are country dependent and some have historical roots, others are taken as a precaution against impulsive breaking wave conditions. Those used in this example, are (see Figure 1)

1. **Layers slopes and berms widths**: The seaside and leeward berm and slope protection has the following restrictions. The minimum armor unit weight allowed is 0.3 kN while the maximum is 21 kN (concrete pieces have to be used for greater weight armor units), this implies that the armor layer thickness limits are ($e = 2\ell_e$):

$$0.5 \le e \le 2\ (m), \quad (70)$$

   where ℓ_e is the equivalent cubic block side for the main layer. The minimum berm widths limits, note that berm widths in Spain are smaller than usual berm widths in Japan are:

$$B_m \ge 2\ell_e; \qquad b \ge 2\ell_e. \quad (71)$$

   The gradient of the slope of the rubble mound is usually set at

$$1.5 \le \cot\alpha_s \le 3; \qquad 1.5 \le \cot\alpha_\ell \le 3. \quad (72)$$

2. **Construction or operational reasons**: The caisson width limits are:

$$10 \le B \le 35\ (m), \quad (73)$$

   while the maximum seaward and the minimum leeward freeboard are

$$h_c = h_n + h_b + h_o - h_{lo} - t_r \le 15; \qquad h_n + h_b - h_{lo} - t_r \le 1\ (m), \quad (74)$$

   respectively, where h_b is caisson height, h_n is core height, h_{lo} is the minimum water depth value in front of the breakwater and t_r is the tidal range.

The minimum water level in front of the vertical breakwater and the minimum water depth in front of the caisson are respectively,

$$h \geq h_{lo}; \qquad d \geq h_{lo} - (h_h + e). \tag{75}$$

The following constraints are used for considering the vertical breakwater as a composite breakwater:

$$\frac{h_n + e}{h_{lo} + t_r} \geq 0.3; \qquad \frac{h_n + e}{h_{lo}} \leq 0.9. \tag{76}$$

For safety reasons the minimum parapet breakwater height is limited to:

$$h_o \geq 1 \ (m). \tag{77}$$

3. **Geometric identities**:

$$h = h_1 + h_2; \qquad h = h' + h_n; \qquad h' + h_c = h_b + h_o; \qquad d + e = h' \tag{78}$$

where h_1 is the water level owing to the astronomical tide, and h_2 is the water level produced by barometrical or storm surge effects.

5.3 Failure rate upper bounds

Thought the different maritime structures can be used in very different conditions where the consequences of a partial or complete failure also are very different, and the accepted probability of failure can vary considerably. Human life, quality and service reliability, and perhaps other criteria must be considered and some constraints on the yearly failure probability rate must be imposed by code designers.

In this paper the selection of the failure rates depends on the consequences of failures, thus, the greater the consequence of failure is the lower failure rate bound is selected. Note that all failure rates are considered annual except the seaward geotechnical failure that will be considered in the whole lifetime of the structure.

The upper failure rate bounds are:

$$\begin{array}{c} R_s^0 = 0.01; \quad R_t^0 = 0.001; \quad R_b^0 = 0.001; \quad R_d^0 = 0.001 \\ R_{sea}^0 = 0.001; \quad R_o^0 = 0.03; \quad R_a^0 = 0.01. \end{array} \tag{79}$$

6 Statistical assumptions

To complete the model, the statistical assumptions need to be provided. They are strongly dependent on the location of the maritime structure. For illustrative purposes, in this section we present those for a composite breakwater in the harbor at Gijón.

6.1 Random and deterministic project factors

The joint distribution of all variables involved is based on the following assumptions (all the numeric values are listed in Table 1):

1. Optimized design variables: The subset $\{B, h_b, h_o\}$ of optimized design variables $\boldsymbol{d}$ related to the concrete caisson are assumed to be deterministic because the construction control is good, whereas the subset of variables associated with the rubble mound $\{b, B_m, e, h_n, \alpha_\ell, \alpha_s\}$ are considered normal random variables whose mean values are obtained from the optimization procedure. In what follows the mean value, standard deviation and the coefficient of variation of any variable x will be denoted as μ_x, σ_x and v_x, respectively.

2. Load variables: The joint distribution of the three-variate random variable ($H_{s_{max}}, H_{max}$, $T_{z_{max}}$) defining our simplified storms and other factors affecting the incident waves, are defined by definition of (see Appendix C for details):

   (a) The marginal cumulative distribution function of $H_{s_{max}}$. Based on extreme value considerations and the truncated character of the simplified storms (they were considered for $H_{s_{max}} \geq 3$), it is shown in Appendix C that $H_{s_{max}}$ can be assumed to be a generalized Pareto distribution.

   (b) The conditional distribution $H_{max}|H_{s_{max}}$ of H_{max} given $H_{s_{max}}$. Based on a regression analysis combined with a probability paper analysis (see Appendix C), we assume $H_{max}|H_{s_{max}}$ to be the maximal Weibull distribution.

   (c) The conditional distribution $T_{z_{max}}|H_{max}, H_{s_{max}}$ of $T_{z_{max}}$ given $H_{max}, H_{s_{max}}$. Based on a regression analysis combined with a probability paper analysis (see Appendix B), we assume that the distribution of $T_{z_{max}}|H_{max}, H_{s_{max}}$ is normal.

   (d) The water depth h_1, considering the tidal elevation, is modelled as a random variable with cumulative distribution function

$$F_{h_1}(x) = \frac{\arccos\left(2(h_{lo} - x)/t_r + 1\right)}{\pi} \tag{80}$$

   where h_{lo} is the minimum value of h_1 (zero port reference level) and t_r is the tidal range.

   (e) The meteorological tide h_2 caused by barometrical effects is assumed to be a normal random variable with mean μ_{h_2} and standard deviation σ_{h_2}.

   (f) The incident wave angle θ_w is assumed to be normal $N(0, \sigma^2_{\theta_w})$.

   (g) The coefficient A in (45) for modelling the change in the maximum wave height due to random wave breaking is modelled as a normal random variable. As there is no clear information on the variance but only reasonable extreme values, the simple rule that two standards deviations account for the difference between the maximum (minimum) and the mean value was adopted. Thus, $\mu_A = (0.18 + 0.12)/2 = 0.15$ and $\sigma_A = (0.18 - 0.12)/4 = 0.015$.

3. The soil strength is modelled using the following assumptions:

   (a) The friction factor μ_c between the caisson base and the rubble is assumed log-normal distributed with mean μ_{μ_c} and standard deviation σ_{μ_c}.

   (b) The friction coefficient μ_s between the rubble bedding layer and the rock subsoil is assumed log-normal distributed with mean μ_{μ_s} and coefficient of variation v_{μ_s}.

   (c) Since the breakwater foundation is made of friction material an statistical model for the angle of internal friction of rubble is required. This angle is modelled by a normal random variable with mean μ_{ϕ_r} and coefficient of variation v_{ϕ_r}. We do not take into account spatial variation.

(d) The average unit weight of caisson γ_c and the unit weight of the rubble γ_s are considered normal random variables with means μ_{γ_c}, μ_{γ_s}, and standard deviations σ_{γ_c}, σ_{γ_s}, respectively.

4. In an attempt to consider all the sources of uncertainty, the uncertainties of the formulas used in the computations have to be examined. Some models are based on empirical relations and show a certain scatter, other are physically based by rely on assumptions or simplifications. In any case a calibration factor is applied to the result of the formula providing the true value.

   (a) The Goda formulae for pulsating wave forces are biased in order to provide a safe relation (see Van der Meer [43] and Oumeraci et al. [37]). The uncertainty is taken into account using the calibration factors A_g, B_g, M_{A_g}, M_{B_g} and S_g affecting horizontal forces (F_h), uplift forces (F_v), horizontal moments (M_h), uplift moments (M_v) and seepage horizontal forces, respectively.

   (b) The reliability of the overtopping prediction formula (62) can be expressed assuming a normal distribution for the random variable b_o, thus $b_o \sim N(\mu_{b_o}, \sigma_{b_o})$ (see Franco and Franco [40]). Note that the coefficient a in (62) is considered deterministic.

   (c) The berm stability function ϕ_e in (64) uncertainty is considered due to the normal random coefficient $C_{ar} \sim N(\mu_{C_{ar}}, \sigma^2_{C_{ar}})$.

5. To consider model uncertainties for the limit state equations model factors equivalent to global safety factors are considered. These will be random parameters F_m (m refers to failure mode) log-normally distributed with expected values μ_{F_m} and coefficients of variation v_{F_m}. Note, for example, that in the overtopping failure, F_o takes into account the uncertainty of the critical structural safety discharge q_0.

All these assumptions and the numeric values used in the example are listed in Table 1.
Dependence assumptions The group of random variables $\{H_{s_{max}}, H_{max}, T_{z_{max}}\}$ are assumed to be dependent with the marginal and conditional distributions given above. For the sake of simplicity, the tidal water level is assumed to be independent of the remaining variables, and the same assumption is used for the meteorological tide; note however that this hypothesis is not really valid because it is dependent on $H_{s_{max}}$. The same would be applicable if storm surge effect in shallow waters were considered.

The remaining variables will be considered independent in this paper (correlation coefficients $\rho_{A_g} = 0$ and $\rho_{B_g} = 0$) though, for example, some authors (see Burcharth and Sorensen [24]) consider dependence between F_h and M_h, and F_v and M_v. It is important to remember here that, in addition, the correlation of the different modes of failure stems from the fact that they depend on common variables that can be dependent or independent. Thus, even in the case of assuming independent variables, the modes of failure will become correlated because of their dependence on common variables. In other words, the main source of mode of failure correlation is its dependence on common variables and not the dependence on the variables themselves.

The above probability functions and the value of their parameters have been chosen solely for illustration purposes. In order to apply the method to real cases, a more careful selection has to be done, using long term data records. Only a few countries have enough information to infer these functions adequately.

Table 1: Statistical model and random model parameters $\boldsymbol{\kappa}$.

i	X_i	Meaning	Mean (μ)	Parameters	Distrib.
1	b	Leeward berm width (m)	$\bar{b}$	$v_b = 0.1$	Normal
2	B_m	Seaward berm width (m)	$\bar{B}_m$	$v_{B_m} = 0.1$	Normal
3	e	Armor protection thickness (m)	$\bar{e}$	$v_e = 0.1$	Normal
4	h_n	Rubble core height (m)	$\bar{h}_n$	$v_{h_n} = 0.1$	Normal
5	α_ℓ	Leeward slope angle (rad)	$\bar{\alpha}_\ell$	$v_{\alpha_\ell} = 0.1$	Normal
6	α_s	Seaward slope angle (rad)	$\bar{\alpha}_s$	$v_{\alpha_s} = 0.1$	Normal
7	$H_{s_{max}}$	Maximum significant wave height (m)		$\kappa_s = -0.1197$ $\delta_s = 0.446$ $\lambda_s = 3$	Pareto
		H_{max} obtained from linear regression of $H_{max}\|H_{s_{max}}$		$a_r = -0.641855$ $b_r = 1.92856$	
8	H_{max}	Residual between the maximum wave height (m) & the one obtained from above		$\kappa_w = 0.172482$ $\delta_w = 0.470151$ $\lambda_w = -0.201646$	Weibull
		$T_{z_{max}}$ obtained from linear regression of $T_{z_{max}}\|H_{max}, H_{s_{max}}$		$a_t = 5.66953$ $b_t = 3.5765$ $c_t = -1.35536$	
9	$T_{z_{max}}$	H_{max} wave period (seg)		$\sigma_{T_{z_{max}}} = 1.6128$	Normal
10	h_1	Tidal water level (m)		$h_{lo} = 20$ $t_r = 5$	Cosine
11	h_2	Meteorological water level (m)	0.02414	$\sigma_{h_2} = 0.11597$	Normal
12	θ_w	Incident wave angle (rad)	0.0	$\sigma_{\theta_w} = \pi/18$	Normal
13	A	Random wave breaking coefficient	0.15	$\sigma_A = 0.015$	Normal
14	μ_c	Friction factor caisson-rubble	0.636	$\sigma_{\mu_c} = 0.0954$	LN
15	μ_s	Friction factor rubble-rock	0.5	$v_{\mu_s} = 0.1$	LN
16	ϕ_r	Rubble friction factor (rad)	0.601	$v_{\phi_r} = 0.1$	Normal
17	γ_c	Average density of caisson (kN/m^3)	22.3	$\sigma_{\gamma_c} = 0.11$	Normal
18	γ_s	Rubble unit weight (kN/m^3)	21	$\sigma_{\gamma_s} = 0.11$	Normal
19	A_g	F_h model uncertainty	0.9	$\sigma_{A_g} = 0.2$	LN
20	B_g	F_v model uncertainty	0.77	$\sigma_{B_g} = 0.2$	LN
21	M_{A_g}	M_h model uncertainty	0.72	$\sigma_{M_{A_g}} = 0.37$	LN
22	M_{B_g}	F_v model uncertainty	0.72	$\sigma_{M_{B_g}} = 0.34$	LN
23	S_g	Seepage model uncertainty	0.65	$\sigma_{S_g} = 0.30$	LN
24	b_o	Overtopping model uncertainty	3	$\sigma_{b_o} = 0.26$	Normal
25	C_{ar}	Stability function uncertainty	1	$\sigma_{C_{ar}} = 0.1$	Normal
	F_m	$m = s, t, b, c, d, sea$	1	$v_{F_m} = 0.2$	LN
32	F_a	Armor failure uncertainty, $m = a$	1	$v_{F_a} = 0.1$	LN
33	F_o	Overtopping failure uncertainty, $m = o$	1	$v_{F_o} = 0.1$	LN

7 Numerical example

The proposed method has been implemented in GAMS (General Algebraic Modelling System) (see Castillo, Conejo, Pedregal, García and Alguacil [44]). GAMS is a software system especially designed for solving optimization problems (linear, non-linear, integer and mixed integer) of small to very large size. All the examples have been solved using the generalized reduce gradient method (for more details see VanderPlaats [45] or Bazaraa, Jarvis y Sherali [46]) that has shown good

Table 2: Fixed deterministic parameters used in the numerical example.

i	X_i	Meaning	Value (μ)	Units
1	a	Structure shape coefficient	0.082	$--$
2	$C_{a\ell}$	Armor layer construction cost per unit volume	70	$\$/m^3$
3	C_c	Sand filled caisson construction cost per unit volume	123	$\$/m^3$
4	C_{co}	Rubble core construction cost per unit volume	2.4	$\$/m^3$
5	q_0	Maximum allowable mean overtopping discharge for structural damage	0.2	$m^3/s/m.l.$
6	R_s^0	Sliding failure rate upper bound, $m = s$	0.01	$--$
	R_m^0	$m = \{t, b, c, d, sea\}$ failure rate upper bound	0.001	$--$
12	R_o^0	Overtopping repair percentage, $m = o$	0.01	$--$
13	R_a^0	Armor failure rate upper bound, $m = a$	0.01	$--$
14	r_{st}	Mean number of storms per year	45.3427	$storms/year$
15	w_o	Caisson parapet width	2	m
16	γ_w	Water unit weight	10.35	kN/m^3
17	$\tan\theta_b$	Mean angle tangent of the sea bottom	1/50	$--$

convergence properties including constraints to the variables. The main advantages of GAMS are:

1. It is a high quality software package (reliable, efficient, fast, widely tested, etc.)
2. It allows the problem to be defined as it is stated mathematically, i.e., without difficult transformations.
3. It allows relations to be handled in implicit or explicit forms.
4. It allows very large (in terms of number of variables or constraints) problems to be solved.
5. Unlike level II methods FORM the proposed method does not need to invert the Rosenblatt transformation and the failure region need not be written in terms of the normalized transformed variables.

Of course, other optimization programs such as AIMMS [47, 48], AMPL [49, 50], LINDO, What's Best, MPL or the Matlab Optimization Toolbox, can be used instead.

To illustrate the method, the automatic optimal design (see Figure 1) of a composite breakwater with the statistical model and random model parameters $\boldsymbol{\kappa}$ and the fixed deterministic parameters shown in Tables 1 and 2, respectively, has been performed. Note that the maximum yearly failure rates have been defined depending on the importance of the corresponding failure.

Analysis of results The following conclusions can be drawn from the analysis of the results:

1. The proposed method leads to the solution of the breakwater design showing a good behavior, the number of reliability evaluations is 72, lower than the the typical number used in these king of problems $(100 - 500)$ (see Voortman et al. [11]). Note that in the computational example we have used 9 design variables, 33 statistical variables and 8 failure modes.
2. Table 3 shows the convergence of the process that is attained after 9 iterations with an error tolerance lower than 3×10^{-6}. The first column shows the initial/construction cost (C_0),

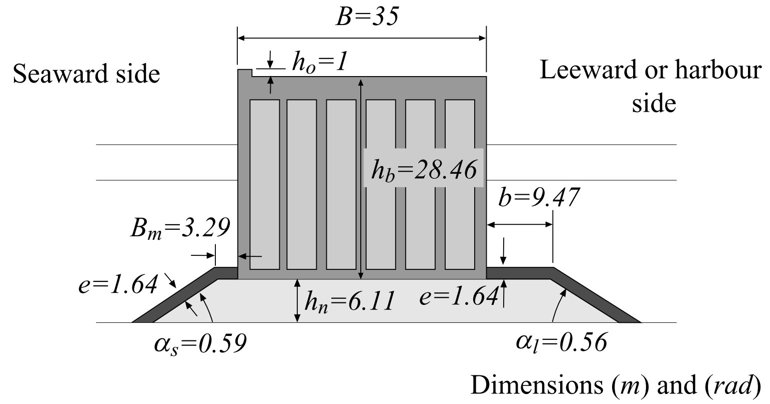

Figure 3: Scaled optimal solution for the vertical breakwater example.

Table 3: Illustration of the convergence of the iterative method.

		ITERATIONS				
Variable	Units	1	2	3	8	9 (end)
C_0	\$	92019.5	104321.8	115128.7	127926.5	**127926.2**
b	m	30.00	32.62	26.79	9.59	**9.47**
B	m	30.00	34.03	34.08	35.00	**35.00**
B_m	m	5.00	3.72	3.06	3.29	**3.29**
e	m	2.00	1.86	1.53	1.65	**1.65**
h_b	m	22.00	22.38	25.69	28.46	**28.46**
h_n	m	7.00	6.97	5.97	6.11	**6.11**
h_o	m	3.00	3.90	2.73	1.00	**1.00**
α_ℓ	m	0.59	0.59	0.59	0.57	**0.56**
α_s	m	0.59	0.59	0.59	0.59	**0.59**
r_s	–	0.00697	0.00376	0.00136	0.00046	**0.00046**
r_t	–	0.00034	0.00009	0.00004	0.00001	**0.00001**
r_b	–	0.00958	0.00384	0.00204	0.00100	**0.00100**
r_c	–	0.00403	0.00191	0.00103	0.00100	**0.00100**
r_d	–	0.00272	0.00145	0.00105	0.00075	**0.00075**
r_{sea}	–	0.00000	0.00000	0.00000	0.00000	**0.00000**
r_o	–	0.08185	0.03614	0.01833	0.01000	**0.01000**
r_a	–	0.03102	0.01161	0.01102	0.01000	**0.01000**

the optimized design variables $\boldsymbol{d}$, and the yearly failure rates of the different failure modes resulting after each iteration. The last column gives the corresponding final values. The scale optimal breakwater design is shown in Figure 3.

3. It is interesting to see that as in the first iterations the failure rate constraints do not hold (they are greater than the maximum failure rates R_m^0), the construction cost is increased in order to increase the safety levels until the final design is obtained, where all the reliability constraints hold.

4. At the optimal solution four reliability constraints are active, rotation failure (b), rupture

Table 4: Failure or maximum likelihood points for each failure mode corresponding to the optimal design.

x_i	$\boldsymbol{z}_0 = \mathbf{0}$	s	t	b	c	d	sea	o	a
b	9.4724	9.4724	9.4724	9.4724	9.4171	9.4426	9.4724	9.4724	9.4724
B_m	3.2909	3.2909	3.2909	3.2909	3.2909	3.2909	3.2401	3.2909	3.4629
e	1.6454	1.6510	1.6498	1.6503	1.6469	1.6482	1.6280	1.6454	1.5108
h_n	6.1123	6.0956	5.9922	6.1060	6.1709	6.0585	7.1086	5.9409	6.2666
α_ℓ	0.5637	0.5637	0.5637	0.5637	0.5676	0.5651	0.5637	0.5637	0.5637
α_s	0.5880	0.5880	0.5880	0.5880	0.5880	0.5880	0.6289	0.5880	0.5880
$H_{s_{max}}$	3.3223	9.4296	7.4917	9.2105	9.2521	9.3759	3.3223	8.9424	8.0122
H_{max}	5.7308	17.4631	13.8981	17.0571	17.1345	17.3602	5.7308	16.5694	14.9467
$T_{z_{max}}$	9.7846	16.5194	14.7098	16.2080	16.2649	16.4660	9.7846	15.1945	13.8174
h_1	22.5000	24.0418	24.0841	23.8242	23.8992	23.9551	22.5000	24.3648	20.7681
h_2	0.0241	0.0277	0.0279	0.0267	0.0270	0.0273	0.0241	0.0303	0.0193
θ_w	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A	0.1500	0.1585	0.1500	0.1570	0.1571	0.1582	0.1500	0.1500	0.1625
μ_c	0.6290	0.5417	0.6290	0.6290	0.6290	0.6290	0.6290	0.6290	0.6290
μ_s	0.4975	0.4975	0.4975	0.4975	0.4975	0.4652	0.4975	0.4975	0.4975
ϕ_r	0.6010	0.6010	0.6010	0.5205	0.5432	0.6007	0.3651	0.6010	0.6010
γ_c	22.3000	22.2941	22.2914	22.2966	22.2951	22.2951	22.3000	22.3000	22.3000
γ_s	21.0000	21.0000	21.0000	20.9972	20.9993	20.9987	21.0000	21.0000	20.9907
A_g	0.8786	1.2141	0.8786	1.0615	1.1634	1.1919	0.8786	0.8786	0.8786
B_g	0.7453	0.7935	0.7453	0.7253	0.7724	0.7825	0.7453	0.7453	0.7453
M_{A_g}	0.6404	0.6404	2.7363	1.0185	0.6865	0.6487	0.6404	0.6404	0.6404
M_{B_g}	0.6511	0.6511	1.2600	0.8749	0.6854	0.6574	0.6511	0.6511	0.6511
S_g	0.5902	0.5902	0.5902	0.5902	0.6044	0.6392	0.5902	0.5902	0.5902
b_o	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	2.8816	3.0000
C_{ar}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0264
F_s	0.9806	1.2758	0.9806	0.9806	0.9806	0.9806	0.9806	0.9806	0.9806
F_t	0.9806	0.9806	1.5182	0.9806	0.9806	0.9806	0.9806	0.9806	0.9806
F_b	0.9806	0.9806	0.9806	1.0786	0.9806	0.9806	0.9806	0.9806	0.9806
F_c	0.9806	0.9806	0.9806	0.9806	1.2382	0.9806	0.9806	0.9806	0.9806
F_d	0.9806	0.9806	0.9806	0.9806	0.9806	1.2773	0.9806	0.9806	0.9806
F_{sea}	0.9806	0.9806	0.9806	0.9806	0.9806	0.9806	0.3849	0.9806	0.9806
F_o	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	1.0092	0.9950
F_a	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	1.0223

surface through rubble only (c), overtopping (o) and armor instability failure (a). Thus, it can be concluded that the ultimate limit state failure of the composite breakwater is determined by subsoil failure as was pointed out by Voortman et al [11], Martinelli et al. [51]. The safety requirements corresponding to the remaining failure modes are ensured with the satisfaction of these four failure modes, this implies, for example, that sliding and overturning failure modes are correlated with then.

5. Table 4 shows the failure points or points of maximum likelihood per failure mode, that is the most probable values of the random variables that induce the failure of the structure. The second column show the most probable values of the random variables ($\boldsymbol{z}_0$) for comparing with the failure points. See for example, that the most probable significant wave height $H_{s_{max}}$ is 3.3223 (m), but the storms which cause the failure of the structure are characterized by design wave conditions with significant wave heights between 7.4917 and 9.4296 (m), maximum wave heights between 13.8981 and 17.4631 (m), and wave periods between 13.8174 and 16.5194 (s) depending on the failure mode, that coincide with the range of design wave conditions which caused disasters in the vertical breakwaters built before World War II (see Oumeraci [52]).

6. The cost sensitivities with respect to the cost of materials and some parameters of the model

Table 5: Sensitivities of the total expected cost with respect $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$ parameters.

X_i	$\frac{\partial E[cost]}{\partial x_i}\lvert x_i\rvert$ (\$)
μ_{h_2}	123.2
μ_{θ_w}	-
μ_A	141887.2
μ_{μ_c}	-
μ_{μ_s}	-
μ_{ϕ_r}	-235770.3
μ_{γ_c}	-138398.5
μ_{γ_s}	-178656.6
μ_{A_g}	72939.1
μ_{B_g}	-7304.3
$\mu_{M_{A_g}}$	30221.0
$\mu_{M_{B_g}}$	25769.2
μ_{S_g}	300.9
μ_{b_o}	-33985.9
$\mu_{C_{ar}}$	14233.7
μ_{F_s}	-
μ_{F_t}	-
μ_{F_b}	45505.5
μ_{F_c}	12597.3
μ_{F_d}	-
$\mu_{F_{sea}}$	-
μ_{F_o}	9207.3
μ_{F_a}	14609.7
a	9207.3
C_{al}	4349.0
C_c	122752.6
C_{co}	824.5
q_0	-9207.3
r_{st}	30948.2
w_o	-427.5
γ_w	316982.4
θ_b	18545.2
R_s^0	-
R_t^0	-
R_b^0	-17858.1
R_c^0	-2018.9
R_d^0	-
R_{sea}^0	-
R_o^0	-6039.9
R_a^0	-5031.4

X_i	$\frac{\partial E[cost]}{\partial x_i}\lvert x_i\rvert$ (\$)
v_b	7.2
v_{B_m}	1473.0
v_e	3622.2
v_{h_n}	873.0
v_{α_ℓ}	10.2
v_{α_s}	-
κ_S	86380.4
δ_S	156116.5
λ_S	78796.3
a_r	-569.0
b_r	8052.8
κ_W	-64.2
δ_W	789.1
λ_W	-178.8
a_T	29653.9
b_T	174599.7
c_T	-122459.2
$\sigma_{T_{zmax}}$	4249.2
h_{lo}	102092.0
t_r	27067.6
σ_{h_2}	38.3
σ_{θ_w}	-
σ_A	8243.9
σ_{μ_c}	-
v_{μ_s}	-
v_{ϕ_r}	35582.4
σ_{γ_c}	22.7
σ_{γ_s}	50.0
σ_{A_g}	12918.6
σ_{B_g}	665.1
$\sigma_{M_{A_g}}$	7502.5
$\sigma_{M_{B_g}}$	2273.3
ρ_{A_g}	15860.7
ρ_{B_g}	-1278.7
σ_{S_g}	-40.1
σ_{b_o}	1341.4
$\sigma_{C_{ar}}$	376.0
v_{Fs}	-
v_{Ft}	-
v_{Fb}	2500.9
v_{Fc}	2397.4
v_{Fd}	-
$v_{F_{sea}}$	-
v_{Fo}	38.6
v_{Fa}	247.6

($\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$) are given in Table 5. It allows one to know how much a small change in a single design factor value changes the optimal expected cost per running meter of the composite breakwater. This information is extremely useful during the construction process to control the cost, and for analyzing how the changes in the yearly failure rates required by the codes influence the total cost of maritime works. For example, a change of one unit in the cost of concrete C_c leads to a relative cost increase of 122752.6 \$ (see the corresponding entry in Table 5). Similarly, while an increase in the unit weight of the rubble mound γ_s decreases the cost (-178656.6 \$), both the tidal range (t_r) and the zero port (h_{lo}) increase the cost by 27067.6 and 102092.0 \$ per relative unit increase, respectively. Note that the most restricted failure

Table 6: Sensitivities of the the yearly failure rates r_m with respect the design variables ($\bar{\boldsymbol{d}}$).

X_i	$\frac{\partial r_s}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_t}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_b}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_c}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_d}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_{sea}}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_o}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_a}{\partial x_i}\lvert x_i\rvert$ (\$)
b	-	-	-	-0.000609	-0.000245	-	-	-
B	-0.003236	-0.000130	-0.010785	-0.007704	-0.004960	-	-	-
B_m	-	-	-	-	-	-	-	0.058918
e	0.000163	0.000002	0.000313	0.000092	0.000130	-	-	-0.080433
h_b	-0.003224	-0.000059	-0.001518	-0.005433	-0.004000	-	-0.139787	-
h_n	-0.000132	-0.000016	-0.000108	0.001015	-0.000682	-	-0.029183	0.027702
h_o	0.000060	0.000002	0.000143	0.000120	0.000094	-	-0.004912	-
α_ℓ	-	-	-	0.000734	0.000197	-	-	-
α_s	-	-	-	-	-	-	-	-

Table 7: Sensitivities of the yearly failure rates r_m with respect the $\tilde{\boldsymbol{\eta}}$ parameters.

X_i	$\frac{\partial r_s}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_t}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_b}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_c}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_d}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_{sea}}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_o}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_a}{\partial x_i}\lvert x_i\rvert$ (\$)
μ_{h_2}	0.000003	-	0.000005	0.000005	0.000004	-	0.000119	-0.000093
μ_{θ_w}	-	-	-	-	-	-	-	-
μ_A	0.002747	-	0.004877	0.004962	0.004282	-	-	0.088997
μ_{μ_c}	-0.003798	-	-	-	-	-	-	-
μ_{μ_s}	-	-	-	-	-0.005261	-	-	-
μ_{ϕ_r}	-	-	-0.012172	-0.009118	-0.000039	-	-	-
μ_{γ_c}	-0.005248	-0.000131	-0.006670	-0.009550	-0.007113	-	-	-
μ_{γ_s}	-	-	-0.005026	-0.001191	-0.001752	-	-	-0.171919
μ_{A_g}	0.002374	-	0.003550	0.004723	0.003701	-	-	-
μ_{B_g}	0.000466	-	-0.000476	0.000592	0.000592	-	-	-
$\mu_{M_{A_g}}$	-	-0.000006	0.001652	0.000357	0.000052	-	-	-
$\mu_{M_{B_g}}$	-	0.000016	0.001409	0.000304	0.000044	-	-	-
μ_{S_g}	-	-	-	0.000149	0.000355	-	-	-
μ_{b_o}	-	-	-	-	-	-	-0.056269	-
$\mu_{C_{ar}}$	-	-	-	-	-	-	-	0.028290
μ_{F_s}	0.003247	-	-	-	-	-	-	-
μ_{F_t}	-	0.000093	-	-	-	-	-	-
μ_{F_b}	-	-	0.002548	-	-	-	-	-
μ_{F_c}	-	-	-	0.006240	-	-	-	-
μ_{F_d}	-	-	-	-	0.005261	-	-	-
$\mu_{F_{sea}}$	-	-	-	-	-	-	-	-
μ_{F_o}	-	-	-	-	-	-	0.015244	-
μ_{F_a}	-	-	-	-	-	-	-	0.029037
a	-	-	-	-	-	-	0.015244	-
$C_{a\ell}$	-	-	-	-	-	-	-	-
C_c	-	-	-	-	-	-	-	-
C_{co}	-	-	-	-	-	-	-	-
q_0	-	-	-	-	-	-	-0.015244	-
r_{st}	0.000465	0.000008	0.001000	0.001000	0.000746	-	0.010000	0.010000
w_o	-0.000011	-	-0.000035	-0.000024	-0.000015	-	-	-
γ_w	0.005247	0.000131	0.011694	0.010739	0.008864	-	-	0.171843
θ_b	0.000373	-	0.000654	0.000668	0.000580	-	-	0.010959

mode is the rotation one (b) because its absolute value sensitivity is the greater (17858.1), obviously, if the maximum yearly failure rate is increased, the cost decrease (negative value of the sensitivity). It is important to mention that as the weakest link in the optimal design is the subsoil failure the relative sensitivity with respect the rubble friction factor (μ_{ϕ_r}) is the greatest in absolute value (-235770.3), thus, it can be concluded that the uncertainty in the subsoil properties plays a very important role (see Voortman et al. ?).

Table 8: Sensitivities of the yearly failure rates r_m with respect $\boldsymbol{\kappa}$ parameters.

X_i	$\frac{\partial r_s}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_t}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_b}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_c}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_d}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_{sea}}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_o}{\partial x_i}\lvert x_i\rvert$ (\$)	$\frac{\partial r_a}{\partial x_i}\lvert x_i\rvert$ (\$)
v_b	-	-	-	0.000004	-	-	-	-
v_{B_m}	-	-	-	-	-	-	-	0.002928
v_e	-	-	-	-	-	-	-	0.007166
v_{h_n}	-	-	-	0.000010	0.000006	-	0.000842	0.000682
v_{α_ℓ}	-	-	-	0.000005	-	-	-	-
v_{α_s}	-	-	-	-	-	-	-	-
κ_S	-0.001395	-0.000040	-0.002906	-0.002928	-0.002228	-	-0.028183	-0.022953
δ_S	0.002389	0.000089	0.005104	0.005118	0.003840	-	0.051111	0.047227
λ_S	0.001115	0.000060	0.002466	0.002456	0.001807	-	0.025803	0.028268
a_r	-0.000056	0.000002	-0.000078	-0.000085	-0.000090	-	-	0.004236
b_r	-0.001599	0.000056	-0.002154	-0.002372	-0.002524	-	-	0.101973
κ_W	-	-	-	-	-	-	-	-0.000145
δ_W	-0.000011	0.000001	-0.000017	-0.000018	-0.000017	-	-	0.002232
λ_W	-0.000018	-	-0.000024	-0.000027	-0.000028	-	-	0.001331
a_T	0.000837	0.000020	0.001636	0.001666	0.001349	-	-	-0.005824
b_T	0.004979	0.000093	0.009507	0.009724	0.007980	-	-	-0.029435
c_T	0.003495	0.000065	0.006672	0.006825	0.005600	-	-	-0.020809
$\sigma_{T_{z_{max}}}$	0.000117	0.000004	0.000207	0.000214	0.000189	-	-	0.000256
h_{lo}	0.002557	0.000047	0.004050	0.004501	0.003629	-	0.098245	-0.076851
t_r	0.000517	0.000010	0.000774	0.000877	0.000718	-	0.021441	-0.002951
σ_{h_2}	-	-	-	-	-	-	0.000030	0.000019
σ_{θ_w}	-	-	-	-	-	-	-	-
σ_A	0.000156	-	0.000227	0.000235	0.000235	-	-	0.007397
σ_{μ_c}	0.000551	-	-	-	-	-	-	-
v_{μ_s}	-	-	-	-	0.000403	-	-	-
v_{ϕ_r}	-	-	0.001883	0.000970	-	-	-	-
σ_{γ_c}	0.000001	-	0.000001	0.000002	0.000002	-	-	-
σ_{γ_s}	-	-	-	-	-	-	-	0.000076
σ_{A_g}	0.000872	-	0.000567	0.001387	0.001238	-	-	-
σ_{B_g}	-0.000001	-	0.000039	-0.000016	-0.000009	-	-	-
$\sigma_{M_{A_g}}$	-	0.000071	0.000425	-0.000046	-0.000009	-	-	-
$\sigma_{M_{B_g}}$	-	0.000011	0.000131	-0.000036	-0.000007	-	-	-
ρ_{A_g}	-	-	0.000866	0.000193	0.000029	-	-	-
ρ_{B_g}	-	-	-0.000074	0.000017	0.000003	-	-	-
σ_{S_g}	-	-	-	-0.000020	-0.000033	-	-	-
σ_{b_o}	-	-	-	-	-	-	0.002221	-
$\sigma_{C_{ar}}$	-	-	-	-	-	-	-	0.000747
v_{F_s}	0.000713	-	-	-	-	-	-	-
v_{F_t}	-	0.000036	-	-	-	-	-	-
v_{F_b}	-	-	0.000140	-	-	-	-	-
v_{F_c}	-	-	-	0.001188	-	-	-	-
v_{F_d}	-	-	-	-	0.001162	-	-	-
$v_{F_{sea}}$	-	-	-	-	-	-	-	-
v_{F_o}	-	-	-	-	-	-	0.000064	-
v_{F_a}	-	-	-	-	-	-	-	0.000492

7. Note that increases of the variable dispersions usually lead to cost increases except for some parameters related to the Goda formulae uncertainty.

8. The sensitivities of the yearly failure rates with respect to the optimized design variables and $\bar{\boldsymbol{d}}$, non-optimized design variables $\tilde{\boldsymbol{\eta}}$ and parameters $\boldsymbol{\kappa}$ are given in tables 6, 7 and 8, respectively. As an example, the influence of the freeboard on the verification equation for overtopping (63) and, therefore, on the corresponding yearly failure rate will be analyzed. This equation shows how this failure occurrence depends only on h_c, q_0, a, b_o and $H_{s_{max}}$. Note that increasing only the freeboard will lead to a safer structure. The freeboard is defined as

$h_c = h_n + h_b + h_o - h$ with $h = h_1 + h_2$ and $h_{lo} \leq h_1 \leq h_{lo} + t_r$. Any increase on variables related to water depth h_{lo}, t_r, h_2 will provide an increase on yearly failure rate for overtopping while any increase on variables related to breakwater heights h_n, h_b, h_o will generate a decrease of yearly failure rate for overtopping due to the fact that all of them appear in the freeboard definition with negative sign for water depths and positive sign for breakwater heights.

8 Conclusions

The methodology presented in this paper, denoted optimal dual method failure-rate versus failure-probability, provides a rational and systematic procedure for automatic and optimal design of maritime works. The engineer is capable of observing simultaneous bounds for the yearly failure rates and probabilities of failure against different modes of failure, so that the most stringent conditions prevail. In addition, a sensitivity analysis can be easily performed by transforming the input parameters into artificial variables, which are constrained to take their associated constant values. The provided example illustrates how this procedure can be applied and proves that it is very practical and useful.

Some additional advantages of the proposed method are:

1. The method allows and easy connection with optimization frameworks.
2. The responsibility for iterative methods is given to the optimization software.
3. The reliability analysis takes full advantage of the optimization packages, which allows the solution of huge problems without the need of being an expert in optimization techniques.
4. Sensitivity values with respect to the target reliability levels are given, without additional cost, by the values of the dual problem.
5. It can be applied to different types of problems such as linear, non-linear, mixed-integer problems. The designer needs just to choose the adequate optimization algorithm.

9 Acknowledgments

The authors are indebted to the Spanish Ministry of Science and Technology (Project DPI2002-04172-C04-02), the Commission of the European Communities in the framework of the EESD Programme under contract no. EVK3-CT-2000-00037 (HUMOR Project), to the Spanish Ministry of Education, Culture and Sports, to the FulBright Commission and to Iberdrola for partial support of this work. The authors also want to thank Antonio Conejo and the ETSII of the University of Castilla-La Mancha for providing some ideas helpful for this paper.

A Appendix: Set of variables

As an illustration, for the composite breakwater example (see Figure 1), the optimized design variables, $\boldsymbol{d}$, are the geometric variables defining the dimensions of the main elements of the composite breakwater, i.e., (see Figure 1):

$$\boldsymbol{d} = \{b, B, B_m, e, h_b, h_n, h_o, \alpha_\ell, \alpha_s\}, \tag{81}$$

the non-optimized design variables, $\boldsymbol{\eta}$, include other geometric variables, costs of materials, unit weights, etc., i.e.:

$$\begin{aligned}\boldsymbol{\eta} &= \{H_{max}, H_{s_{max}}, T_{z_{max}}, h_1, h_2, \theta_w, A, \mu_c, \mu_s, \phi_r, \gamma_c, \gamma_s, A_g, B_g, M_{A_g}, M_{B_g}, S_g, b_o, C_{ar}\} \\ &\quad \cup \{F_m, a, C_{a\ell}, C_c, C_{co}, q_0, r_{st}, w_o, \gamma_w, \tan\theta_b, R_m^0\},\end{aligned}$$

the random model parameters, $\boldsymbol{\kappa}$, include the coefficients of variations, standard deviations, parameters of the joint probability density function, etc.:

$$\begin{aligned}\boldsymbol{\kappa} &= \{v_b, v_{B_m}, v_e, v_{h_n}, v_{\alpha_\ell}, v_{\alpha_s}, \kappa_s, \delta_s, \lambda_s, a_r, b_r, \kappa_w, \delta_w, \lambda_w, a_t, b_t, c_t, \sigma_{T_{z_{max}}}, h_{lo}, t_r, \sigma_{h_2}, \sigma_{\theta_w}\} \\ &\quad \cup \{\sigma_A, \sigma_{\mu_c}, v_{\mu_s}, v_{\phi_r}, \sigma_{\gamma_c}, \sigma_{\gamma_s}, \sigma_{A_g}, \sigma_{B_g}, \sigma_{M_{A_g}}, \sigma_{M_{B_g}}, \rho_{A_g}, \rho_{B_g}, \sigma_{S_g}, \sigma_{b_o}, \sigma_{C_{ar}}, v_{F_m}\},\end{aligned}$$

and the dependent variables, $\boldsymbol{\psi}$, include redundant geometric variables, volumes, moments, etc., that can be written in terms of variables $\boldsymbol{d}$ and $\boldsymbol{\eta}$:

$$\begin{aligned}\boldsymbol{\psi} &= \{B_z, c, d, F_h, F_v, F_{hu}, h, h_0, h', h_c, H_{break}, H_d, \ell_e, L, L_0, M_h, M_v, p_1, p_3, p_4, p_u\} \\ &\quad \cup \{q, R, V_c, V_{a\ell}, V_{co}, W, W_1, y, y_{F_h}, y_{F_v}, \theta_s, \Phi_e\}.\end{aligned}$$

B Appendix: Cost function

Consider the composite breakwater in Figure 1. To derive the cost function the following parts are considered:

Concrete volume: The caisson volume is

$$V_c = Bh_b + w_o h_o \tag{82}$$

Armor layer volume: The armor layer volume is

$$V_{a\ell} = e[B_m + b + h_n(1/\sin\alpha_s + 1/\sin\alpha_\ell) + 0.5e(1/\tan\alpha_s + 1/\tan\alpha_\ell)] \tag{83}$$

Core volume: The core volume is

$$V_{co} = h_n(b + B + B_m - e(\tan(\alpha_s/2) + \tan(\alpha_\ell/2)) + h_n(1/\tan\alpha_s + 1/\tan\alpha_\ell)/2) \tag{84}$$

Then, the construction cost per unit length becomes

$$C_0 = C_c V_c + C_{a\ell} V_{a\ell} + C_{co} V_{co} \tag{85}$$

C Appendix: Statistical definition of Storms

In this appendix we derive the joint distribution of $(H_{s_{max}}, H_{max}, T_{z_{max}})$ based on Gijón buoy data. The data correspond to 5.69 years of observations.

The analysis was done as follows:

1. First, data record with a one missed data point were completed by interpolation. This led to recover 6 data records and discarded only three incomplete and not extreme storms.

2. A threshold level of $H_s = 3$ m was selected and a storm defined for the duration of wave height conditions above this level without going down. This led to 258 storms (with significant wave height > 3m) which implies a yearly rate $r_{st} = 45.3427$ storms/year.

3. The peak value $H_{s_{max}}$ of H_s and the pair maximum observed wave height H_{max} and associated zero-up-crossing mean period $T_{z_{max}}$ during each storm were registered. This means that a sample of three values $H_{s_{max}}, H_{max}$ and $T_{z_{max}}$ per storm was used.

From these data, the joint distribution of the three-variate random variable $(H_{s_{max}}, H_{max}, T_{z_{max}})$ need to be defined. Instead of defining the joint density or cumulative distribution function of $(H_{s_{max}}, H_{max}, T_{z_{max}})$, that is difficult to visualize, without loss of generality, we define:

1. The marginal distribution of $H_{s_{max}}$.

2. The conditional distribution $H_{max}|H_{s_{max}}$ of H_{max} given $H_{s_{max}}$.

3. The conditional distribution $T_{z_{max}}|H_{max}, H_{s_{max}}$ of $T_{z_{max}}$ given $H_{max}, H_{s_{max}}$.

The selection of the adequate marginal and conditional distributions is based on extreme value theory and probability paper techniques (see Castillo [53]), as shown below.

C.1 Marginal distribution of $H_{s_{max}}$

To make a proper selection of the marginal distribution of $H_{s_{max}}$ and since the data come from left truncation at 3 m. of a sample of maxima, first we use theoretical considerations that lead to a maximal generalized Pareto distribution (GPD_M) with cumulative distribution function:

$$F_{H_{s_{max}}}(H_{s_{max}}) = 1 - \left(1 - \frac{\kappa_s(H_{s_{max}} - \lambda_s)}{\delta_s}\right)^{1/\kappa_s} \;;\; 1 - \frac{\kappa_s(H_{s_{max}} - \lambda_s)}{\delta_s} \geq 0. \tag{86}$$

Thus, we have fitted a GPD_M by least squares and obtained the following estimates:

$$\hat{\kappa}_s = -0.1197; \quad \hat{\delta}_s = 0.446; \quad \hat{\lambda}_s = 3.$$

To check the goodness of the model we have plotted the data on P-P and Q-Q plots, as shown in Figure 4. The plots show a reasonable fit, so that based on theory and data evidence we accept the model (86).

C.2 Conditional distribution of H_{max} given $H_{s_{max}}$

To choose a conditional distribution $H_{max}|H_{s_{max}}$ of H_{max} given $H_{s_{max}}$, we first plot the data $(H_{s_{max}}, H_{max})$ and observe that they exhibit a linear regression (see Figure 5):

$$H_{max} = a_r + b_r\, H_{s_{max}}, \tag{87}$$

where the estimated parameters are $\hat{a}_r = -0.641855$ and $\hat{b}_r = 1.92856$.

Next, we calculate the residuals and find that they follow a maximal Weibull model (see the Maximal Weibull probability plot in Figure 6):

$$F_X(x) = \exp\left\{-\left[1 - \kappa_w\left(\frac{x - \lambda_w}{\delta_w}\right)\right]^{1/\kappa_w}\right\}; \quad 1 - \kappa_w\left(\frac{x - \lambda_w}{\delta_w}\right) \geq 0. \tag{88}$$

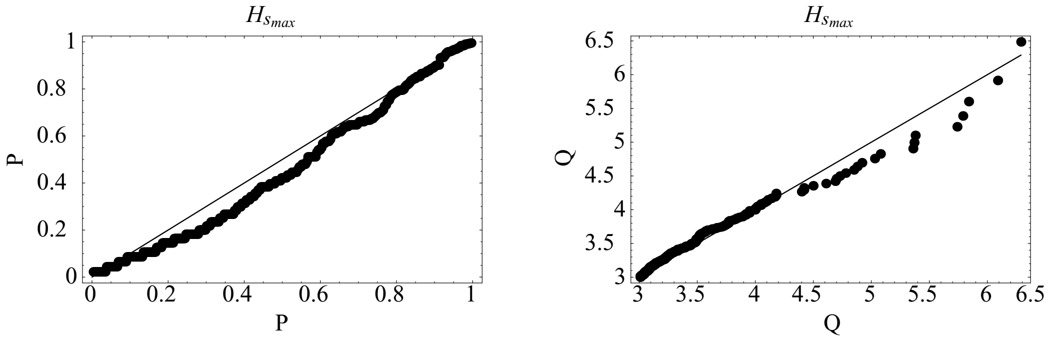

Figure 4: P-P and Q-Q plots corresponding to the maximal generalized Pareto model with cdf in (86).

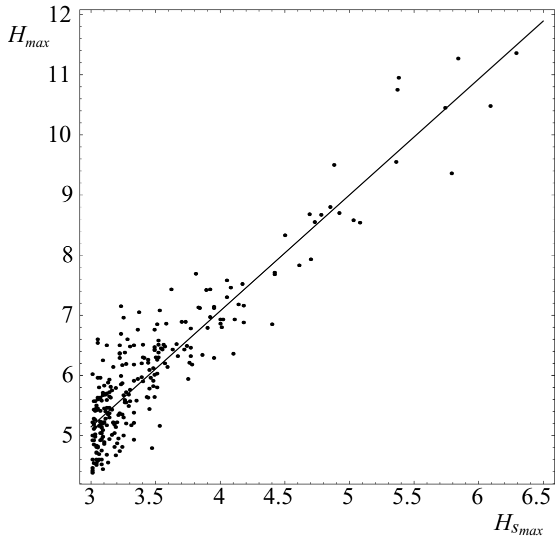

Figure 5: Data $(H_{s_{max}}, H_{max})$ and regression line for the Gijón buoy resulting storms.

Combining this expression with the regression equation (87) leads to the final model for $H_{max}|H_{s_{max}}$

$$F_{H_{max}|H_{s_{max}}=y}(x|y)\exp\left\{-\left[1-\kappa_w\left(\frac{x-a_r-b_s y-\lambda_w}{\delta_w}\right)\right]^{1/\kappa_H}\right\};1-\kappa_w\left(\frac{x-a_r-b_s y-\lambda_w}{\delta_w}\right)\geq 0. \tag{89}$$

Then, estimation of the Weibull parameters using the maximum likelihood method leads to

$$\hat{\kappa}_w = 0.172482; \quad \hat{\delta}_w = 0.470151; \quad \hat{\lambda}_w = -0.201646.$$

C.3 Conditional distribution of $T_{z_{max}}$ given H_{max}, and $H_{s_{max}}$

To derive the conditional distribution $T_{z_{max}}|H_{max}, H_{s_{max}}$ of $T_{z_{max}}$ given $H_{max}, H_{s_{max}}$, we first tried several regression models for $T_{z_{max}}$ given $H_{max}, H_{s_{max}}$ and find as the best model

$$T_{z_{max}} = a_t + b_t\ H_{s_{max}} + c_t\ H_{max}, \tag{90}$$

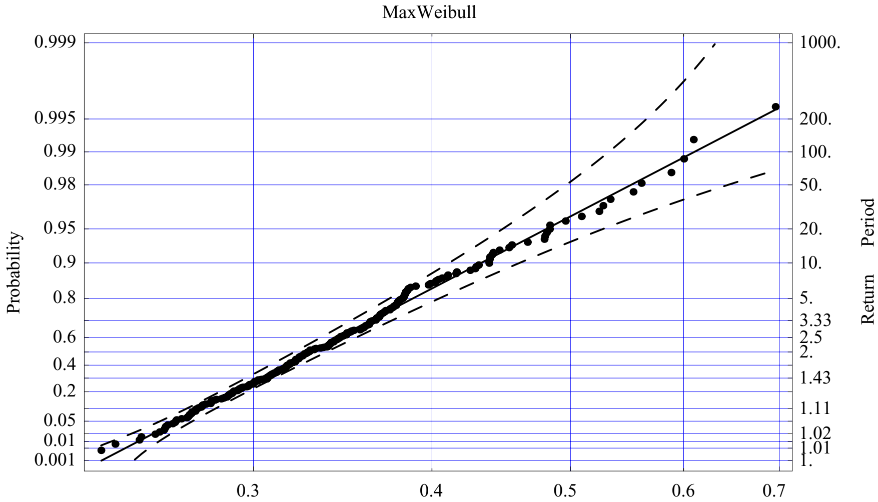

Figure 6: H_{max} residuals given $H_{s_{max}}$ on a Maximal Weibull probability plot.

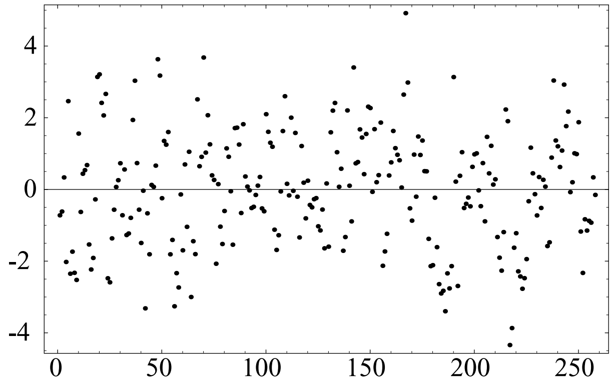

Figure 7: Plot of $T_{z_{max}}$ residuals of the regression equation (90).

where the estimated parameters are $\hat{a}_t = 5.66953$ and $\hat{b}_t = 3.5765$ and $\hat{c}_t = -1.35536$.

Next, we obtain the residuals $\epsilon_i; i = 1, \ldots, 258$ (see Figure 7) and plot them on a normal probability paper, obtaining the plot in Figure 8, that confirms their normality. Once estimated the corresponding parameters by maximum likelihood we get

$$\epsilon_i \sim N(0, \sigma^2_{T_{z_{max}}}),$$

where $\sigma_{T_{z_{max}}} = 1.6128$ that leads to the final model for $T_{z_{max}}|H_{max}, H_{s_{max}}$:

$$T_{z_{max}}|H_{max}, H_{s_{max}} \sim N\left(a_t + b_t\ H_{s_{max}} + c_t\ H_{max}, \sigma^2_{T_{z_{max}}}\right). \tag{91}$$

D Simulation of random storms

If one is interested in simulating random storms, i.e., random values of $(H_{s_{max}}, H_{max}, T_{z_{max}})$, for example to run a Monte Carlo simulation, one can use the following algorithmic process:

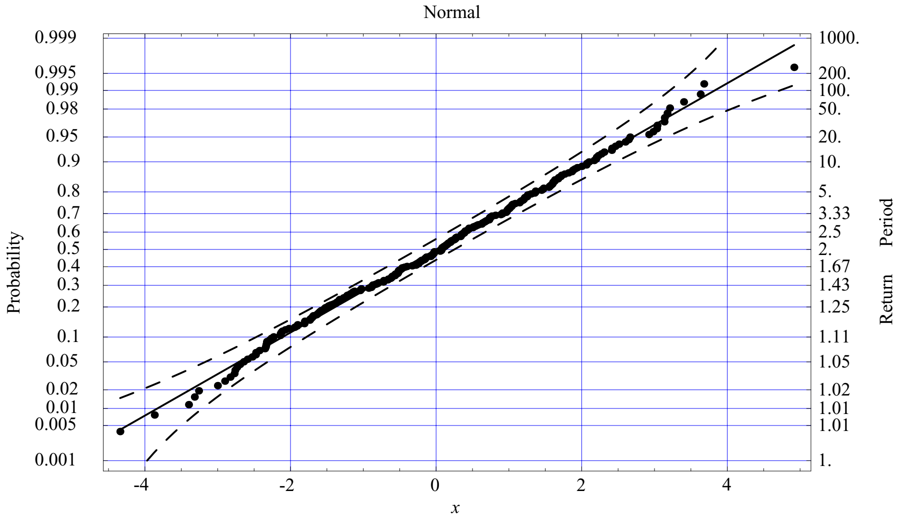

Figure 8: Plot of the $T_{z_{max}}$ residuals of the regression equation (90) on a normal probability plot.

1. Simulate $H_{s_{max}}$ using
$$H_{s_{max}} = \lambda_S + \frac{\delta_S}{\kappa_S}\left(1-(1-u_1)^{\kappa_S}\right),$$
where u_1 is a random uniform $U(0,1)$ number.

2. Simulate H_{max} using
$$H_{max} = a_r + b_r\ H_{s_{max}} + \lambda_w + \frac{\delta_w}{\kappa_w}\left(1-(-\log(u_2))^{\kappa_w}\right),$$
where u_2 is a random uniform $U(0,1)$ number independent of u_1.

3. Simulate $T_{z_{max}}$ using
$$T_{z_{max}} = a_t + b_t\ H_{s_{max}} + c_t\ H_{max} + v$$
where v is a random normal $N(0, 1.61282^2)$ number,

The process must be repeated as many times as the number of desired storms (sample size).

References

[1] M. A. Losada. Recent development in the design of mound breakwaters. In J.B. Herbich, editor, *Chapter 21 in Handbook of Coastal and Ocean Engineering*, volume I. Gulf Publishing, 1990.

[2] Puertos del Estado. Rom 0.0, procedimiento general y bases de cálculo en el proyecto de obras marítimas y portuarias. Technical report, Puertos del Estado, Madrid, España, Noviembre 2001. pp 245.

[3] J. Galambos. *The Asymptotic Theory of Extreme Order Statistics.* Robert E. Krieger Publishing Company, Malabar, Florida, 1987.

[4] E. Castillo. *Extreme Value Theory in Engineering.* Academic Press, New York, 1988.

[5] C. Castillo, M. Losada, R. Mínguez, and E. Castillo. Técnicas de optimización aplicadas al diseño de obras marítimas. In *Procedimiento Metodológico Participativo para la Canalización, Recogida y Difusión de Estudios y Análisis Técnico-Científicos sobre los Documentos del Programa ROM, EROM 00*, EROM. Puertos del Estado, Ministerio de Fomento, 2003.

[6] C. Castillo, M. A. Losada, E. Castillo, and R. Mínguez. Técnicas de optimización aplicadas al diseño de obras marítimas. In *VII Jornadas de Ingeniería de Costas y Puertos*, pages 27–30. Almería, España, 2003.

[7] E. Castillo, R. Mínguez, A. Ruíz-Terán, and A. Fernández-Canteli. Design and sensitivity analysis using the probability-safety-factor method. An application to retaining walls. *Structural Safety*, 26:159–179, 2003.

[8] E. Castillo, M. Losada, R. Mínguez, C. Castillo, and A. Baquerizo. An optimal engineering design method that combines safety factors and failures probabilities: Application to rubble-mound breakwaters. *Journal of Waterways, Ports, Coastal and Ocean Engineering, ASCE*, 130(March):77–88, 2004.

[9] E. Castillo, A. Conejo, R. Mínguez, and C. Castillo. An alternative approach for addressing the failure probability-safety factor method with sensitivity analysis. *Reliability Engineering and System Safety*, 82:207–216, 2003.

[10] D. Van Dantzig. Economic decisiones problems for flood prevention. *Econometrika*, 24:276–287, 1956.

[11] H. G. Voortman, H. K. T. Kuijper, and J. K. Vrijling. Economic optimal design of vertical breakwaters. In *Proceedings of the 26th International Coastal Engineering Conference (ICCE)*, volume 2, pages 2124–2137, Copenhagen, Denmark, 1998. ASCE.

[12] I. Enevoldsen. *Reliability-based structural optimization.* PhD thesis, University of Aalborg, Aalborg, Denmark, 1991.

[13] I. Enevoldsen and J. D. Sorensen. Reliabily-based optimization of series systems of parallel systems. *Journal of Structural Engineering*, 119(4):1069–1084, 1993.

[14] I. Enevoldsen and J. D. Sorensen. Reliabily-based optimization in structural engineering. *Structural Safety*, 15:169–196, 1994.

[15] R. Mínguez, E. Castillo, C. Castillo, and M. Losada. Optimal cost design with sensitivity analysis using decomposition techniques. application to composite breakwaters. *Structural Safety, ASCE*, 2004. Under review.

[16] S. R. K. Nielsen and H. F. Burcharth. Stochastic design of rubble mound breakwaters. In *Proceedings of the 11th IFIP Conference on System Modelling and Optimization*, Copenhagen, Denmark, 1983.

[17] A. M. Hasofer and N. C. Lind. Exact and invariant second moment code format. *J. Engrg. Mech.*, 100(EM1):111–121, 1974.

[18] H. D. Madsen, S. Krenk, and N. C. Lind. *Methods of structural safety.* Prentice Hall, Inc., Englewood Cliffs, New York, second edition edition, 1986.

[19] O. Ditlevsen and H. O. Madsen. *Structural reliability methods.* Wiley, Chichester, New York, 1996.

[20] R. E. Melchers. *Structural reliability analysis and prediction.* John Wiley & Sons, New York, second edition edition, 1999.

[21] H. F. Burcharth. Reliability evaluation of a structure at sea. In *Proc. Int. Workshop on wave barriers in deep water*, Yokosuka, Japan, 1994. Port and Harbour Research Institute.

[22] H. F. Burcharth. Reliability based design of coastal structures. In Philip Liu, editor, *Advances in Coastal and Ocean Engineering*, volume 3, Singapore, 1997. World Scientific Publishing.

[23] H. F. Burcharth. Reliability based design of coastal structures. In *Chapter 6 in Coastal Engineering Manual Part VI (CEM).* Coastal Engineering Research Center, Vicksburg, Mis., U.S., 2000.

[24] H. F. Burcharth and J. D. Sorensen. Design of vertical wall caisson breakwaters using partial safety factors. In *Proceedings of the 25th International Coastal Engineering*, volume 2, Copenhagen, Denmark, 1998.

[25] Y. Goda. *Random Seas and Design of Maritime Structures.* Tokyo University Press, Tokyo, 1985.

[26] Y. Goda and H. Takagi. A reliability design method of caisson breakwaters with optimal wave heights. *Coastal Engineering Journal*, 42(4):357–387, 2000.

[27] E. Christiani. *Application of Reliability in Breakwater Design.* PhD thesis, Hydraulic and Coastal Engineering Laboratory, Dpeartment of Civil Engineering, University of Aalborg, Aalborg, Denmark, 1997.

[28] J. D. Sorensen, H.F. Burcharth, and E. Christiani. Reliability analysis and optimal design of monolithic vertical wall breakwaters. In *Proceedings 6th IFIP WG7.5 Working Conf. on Reliability and Optimization of Structural Systems*, Assissi, Italy, 1994. Chapman & Hall.

[29] J. F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4:238–252, 1962.

[30] A. M Geoffrion. Generalized benders decomposition. *JOTA*, 10(4):237–260, 1972.

[31] R. Mínguez. *Seguridad, Fiabilidad y Análisis de Sensibilidad en Obras de Ingeniería Civil Mediante Técnicas de Optimizaciøn por Descomposiciøn. Aplicaciones.* PhD thesis, University of Cantabria, Santander, Spain, 2003.

[32] R. Mínguez, E. Castillo, and A. S. Hadi. Solving the inverse reliability problem using decomposition techniques. *Structural Safety, ASCE*, 2003. Accepted.

[33] I. Enevoldsen. Sensitivity analysis of reliability-based optimal solution. *Journal of Engineering Mechanics*, 120(1):198–205, 1994.

[34] J. D. Sorensen and I. Enevoldsen. Sensitivity analysis in reliability-based shape optimization. In B. H. V. Topping, editor, *NATO ASI*, volume 221 of *E*, pages 617–638, Dordrecht, The Nethelands, 1992. Kluwer Academic Publishers.

[35] E. Christiani, H. F. Burchart, and J. D. Sorensen. Reliability based optimal design of vertical breakwaters modelled as a series system of failure. In *Proceedings of the 25th International Coastal Engineering Conference (ICCE)*, volume 2, pages 1589–1602, Orlando, Florida (USA), 1996. ASCE.

[36] J. D. Sorensen and H. F. Burcharth. Reliability analysis of geotechnical failure modes for vertical wall breakwaters. *Computers and Geotechnics*, 26:225–245, 2000.

[37] H. Oumeraci, A. Kortenhaus, W. Allsop, M. de Groot, R. Crouch, H. Vrijling, and H. Voortman. *Probabilistic Design Tools for Vertical Breakwaters.* Balkema Publishers, New York, 2001.

[38] Working Group 28 PIANC. Breakwaters with vertical and inclined concrete walls. Report, 2003.

[39] S. Takahashi, K. Tanimoto, and S Shimosako. Experimental study of impulsive pressures on composite breakwaters. Technical Report 5, Port and Harbour Research Institute, 1992.

[40] C. Franco and L. Franco. Overtopping formulas for caissons, breakwaters with nonbreaking 3d waves. *Journal of Waterway, Port, Coastal and Ocean Engineering*, 125(2):98–108, 1999.

[41] L. Franco, M. De Gerloni, and J. W. Van der Meer. Wave overtopping at vertical and composite breakwaters. In *Proceedings of the 24th International Coastal Engineering Conference (ICCE)*, pages 1030–1045, New York, 1994. ASCE.

[42] K. Tanimoto, T. Yagyu, and Y. Goda. Irregular wave tests for composite breakwater foundation. In *Proceedings of the 18th Conference on Coastal Engineering*, pages 2144–2163, Capetown, 1982.

[43] D.J. Van Der Meer, K. D'Angremond, and J. Juhl. Uncertainty on Goda formula. In *Proceedings of the 24th International Coastal Engineering*, Kobe, Japan, 1994.

[44] E. Castillo, A. Conejo, P. Pedregal, R. García, and N. Alguacil. *Building and Solving Mathematical Programming Models in Engineering and Science.* John Wiley & Sons Inc., New York, 2001. Pure and Applied Mathematics: A Wiley-Interscience Series of Texts, Monographs and Tracts.

[45] G. N. Vanderplaats. *Numerical Optimization Techniques for Engineering Design.* McGraw-Hill, New York, 1984.

[46] M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali. *Linear Programming and Network Flows.* John Wiley & Sons, New York, second edition edition, 1990.

[47] http://www.aimms.com/.

[48] J. Bisschop and M. Roelofs. *AIMMS, The User's guide.* Paragon Decision Technology, 1999.

[49] R. Fourer, D. M. Gay, and B. W. Kernighan. *AMPL: A Modelling Language for Mathematical Programming.* boyd & fraser publishing company, Massachusetts, 1993.

[50] http://www.dashopt.com.

[51] L. Martinelli, H. G. Voortman, J.K. Vrijlind, and A Lamberti. Application of probabilistic design tools for vertical breakwaters. In I. J. Losada, editor, *Proceedings of the International conference Coastal Structures 99*, volume 1, Spain, 1999. Balkema.

[52] H. Oumeraci. Review and analysis of vertical breakwater failures- lesson learned. *Coastal Engineering*, (22):3–29, 1994.

[53] E. Castillo, A. S. Hadi, N. Balakrishnan, and J. M. Sarabia. *Extreme Value and Related Models in Engineering and Science Applications.* John Wiley & Sons, New York, 2004.

E Appendix C: Notation

a: Coefficient used in the overtopping formula (62) that depends on the structure shape and on the water surface behavior at the seaward face.

a_r: Linear regression coefficient between $H_{S_{max}}$ and H_{max}.

a_t: Linear regression coefficient between $T_{z_{max}}$ and $H_{S_{max}}$,H_{max}.

A: Coefficient in (45) for modelling the change in the maximum wave height due to random wave breaking.

A_g: model uncertainty of the horizontal forces for the Goda pressure formula.

b: leeward berm width.

b_o: Coefficient used in the overtopping formula (62) that depends on the structure shape and on the water surface behavior at the seaward face.

b_r: Linear regression coefficient between $H_{S_{max}}$ and H_{max}.

b_t: Linear regression coefficient between $T_{z_{max}}$ and $H_{S_{max}}$,H_{max}.

B: Breakwater caisson width.

B_g: model uncertainty of the vertical forces for the Goda pressure formula.

B_m: seaside berm width.

B_z: distance of the resultant vertical force component to the harbor side edge.

c: Auxiliary variable defined in (68).

c_t: Linear regression coefficient between $T_{z_{max}}$ and $H_{S_{max}}$,H_{max}.

$C_{a\ell}$: cost of the armor layer per unit volume.

C_{ar}: model uncertainty for the overtopping formula.

C_c: cost of the concrete per unit volume.

C_{co}: cost of the core per unit volume.

d: berm depth in front of the caisson.

$\boldsymbol{d}$: design or geometric variables.

e: armor layer thickness.

F_h: horizontal force due to water pressure.

F_{hu}: horizontal seepage force on the rubble.

F_m: Model uncertainty parameters related to the different failure modes $m = \{s, t, b, c, d, sea, o, a\}$.

F_v: vertical force due to water pressure.

g: Acceleration of gravity.

h: design water level.

h_0: water height at five times $H_{s_{max}}$ from the breakwater.

h_1: water level owing to the astronomical tide.

h_2: water level owing to the barometrical or storm surge effects.

h_b: breakwater caisson height.

h_c: seaward freeboard.

h_{lo}: zero port reference level.

h_n: core height.

h_o: crownwall parapet height.

h': is the submerged height of the crownwall.

H_{break}: maximum wave height by breaking conditions.

H_{max}: Design wave height.

H_d: Design wave height.

H_{max}: Maximum wave height.

$H_{s_{max}}$: maximum significant wave height.

ℓ_e: equivalent cubic block side.

L: wave length.

L_0: deep water wave length.

M_e: moment of the forces acting on the foundation.

M_h: moment with respect to O of the horizontal water pressure forces.

M_v: moment with respect to O of the vertical water pressure forces.

p_1: wave pressure at the water level.

p_3: wave pressure at the caisson's bottom level.

p_4: wave pressure at the freeboard level.

p_{st}^m: Yearly failure rate for mode $m = \{s, t, b, c, d, sea, o, a\}$.

p_u: Uplift pressure on the base of the crownwall.

q: mean overtopping volume per unit breakwater length.

q_0: maximum mean overtopping volume per unit breakwater length allowed.

r_{st}: Mean number of storms per year.

R: dimensionless constant depending on γ_c and γ_w.

R_m^0: upper bound of yearly failure rate per failure mode $m = \{s, t, b, c, d, sea, o, a\}$.

r_{st}: storm yearly rate.

r_m: yearly failure rate for mode $m = \{s, t, b, c, d, sea, o, a\}$.

S_g: model uncertainty of the seepage horizontal forces for the Goda pressure formula.

t_r: tidal range.

$T_{z_{max}}$: wave period related to the maximum wave height.

u_1, u_2: standard uniform random variables.

v: standard normal random variable.

V_c: caisson concrete sand filled volume.

$V_{a\ell}$: armor layer total volume.

V_{co}: core total volume.

v_x: variation coefficient of the random variable X.

w_o: caisson parapet width.

W: individual armor block weight.

W_1: crownwall weight.

y: offset of W_1.

y_{F_h}: offset of F_h.

y_{F_v}: offset of F_v.

α_ℓ: leeward slope angle.

α_s: seaward slope angle.

β_m: reliability factor for mode $m = \{s, t, b, c, d, sea, o, a\}$.

γ_c: concrete unit weight.

γ_s: rubblemound unit weight.

γ_w: water unit weight.

δ_S: scale parameter of the Pareto distribution for $H_{S_{max}}$.

δ_w: scale parameter of the Weibull distribution for H_{max}.

$\boldsymbol{\eta}$: non-optimized design variables.

$\tilde{\boldsymbol{\eta}}$: mean or characteristic value of $\boldsymbol{\eta}$.

θ_b: mean angle of the sea bottom.

θ_s: angle between the bottom of the sea and the rupture surface for failure mode c.

θ_w: Incidence wave angle.

$\boldsymbol{\theta}$: Parametric vector.

κ_S: shape parameter of the Pareto distribution for $H_{S_{max}}$.

κ_w: shape parameter of the Weibull distribution for H_{max}.

$\boldsymbol{\kappa}$: the set of parameters associated with the random variability and dependence structure of the random variables involved.

λ_S: shape parameter of the Pareto distribution for $H_{S_{max}}$.

λ_w: shape parameter of the Weibull distribution for H_{max}.

μ_c: friction factor between concrete structure and rubble foundation.

μ_s: friction factor between rubble foundation and rock sea bottom.

μ_x: mean value of the random variable X.

ρ_{A_g}: correlation coefficient between F_h and M_h.

ρ_{B_g}: correlation coefficient between F_v and M_v.

σ_x: standard deviation of the random variable X.

ϕ_r: angle of internal friction of rubble.

$\boldsymbol{\psi}$: the auxiliary (non-basic) variables the values of which can be obtained from those of the basic variables.

$\tilde{\boldsymbol{\phi}}$: mean value of $\boldsymbol{\phi}$.

Φ_e: stability function.