An Alternative Approach for Addressing the Failure Probability-Safety Factor Method with Sensitivity Analysis

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Abstract

The paper introduces a method for solving the failure probability-safety factor problem for designing engineering works proposed by Castillo et al., that optimizes an objective function subject to the standard geometric and code constraints, and two more sets of constraints that simultaneously guarantee given safety factors and failure probability bounds associated with a given set of failure modes. The method uses the dual variables and is especially convenient to perform a sensitivity analysis, because sensitivities of the objective function and the reliability indices can be obtained with respect to all data values. To this end, the optimization problems are transformed into other equivalent ones, in which the data parameters are converted into artificial variables, and locked to their actual values. In this way, some variables of the associated dual problems become the desired sensitivities. In addition, using the proposed methodology, calibration of codes based on partial safety factors can be done. The method is illustrated by its application to the design of a simple rubble mound breakwater and a bridge crain.

Key Words: Sensitivity analysis, Optimization, Automatic design, Duality.

1 Introduction and motivation

Engineering design of structural elements is a complicated and highly iterative process that usually requires a long experience. Iterations consists of a trial-and-error selection of the design variables or parameters, together with a check of the safety and functionality constraints, until reasonable structures, in terms of cost and safety, are obtained.

Optimization procedures are a good solution to free the engineer from the above mentioned cumbersome iterative process, i.e., to automate the design process (see Adeli [1], Sarma and Adeli [19, 20], Bazaraa et al. [2], Castillo et al. [6], Luenberger [14], etc.).

Safety of structures is the fundamental criterion for design (see Blockley [3], Ditlevsen and Madsen [9], Eurocode [10], ROM [17], Freudenthal [11], Madsen et al. [15], Melchers

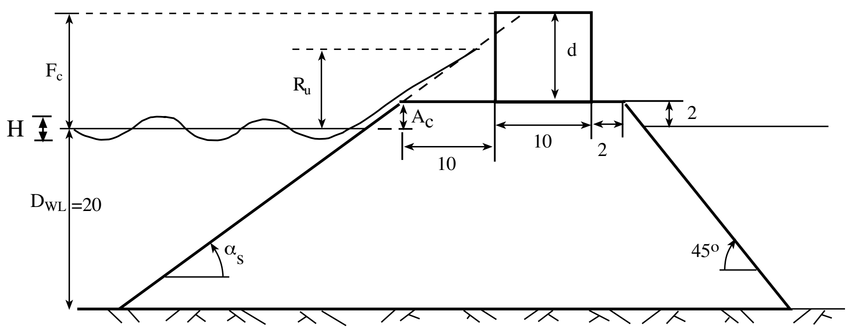

Figure 1: Parameterized rubblemound breakwater used in the example.

[16], Steward and Melchers [21], Wirsching and Wu [22], Wu, Burnside and Cruse [23]). To this end, the engineer first identifies all failure modes of the work being designed and then establishes the safety constraints to be satisfied by the design variables. To ensure satisfaction of the safety constraints, two approaches are normally used: (a) the classical safety factor approach, and (b) the probability based approach.

With the purpose of illustration, consider the case of designing a breakwater (see Figure 1) fixing its geometry and dimensions, and checking its behavior with respect to the most important failure modes, as overtopping, overturning and sliding. This check can be done using safety factors, failure probabilities or both. Each failure mode has a probability of occurrence that depends on the selected geometry. A given design must guarantee that the failure probabilities associated with all failure modes are smaller than the values required by the engineering codes. In addition, it is fundamental to choose a design that minimizes the cost.

Classic engineers critizice the probabilistic approach because of its sensitivity to statistical hypotheses, especially tail assumptions (see Galambos [12] and Castillo [5]). Similarly, Probability based engineers question classical designs because it is not clear how far are their designs from failure. To avoid the lack of agreement between defenders of both approaches, and to obtain a more reliable design, Castillo et al. [7, 8] proposed a mixed method, the failure probability-safety factor method (FPSF) that combines safety factors and failure probability constraints.

Since the failure probability bounds cannot be directly imposed in the form of standard constraints, optimization packages cannot deal directly with problems involving them. In fact, failure probability constraints require themselves the solution of other optimization problems.

Fortunately, there are some iterative methods for solving this problem that converge in a few iterations to the optimal solution (see for example [7, 8]). However, since the proposed method consists of a bilevel minimization process, one that minimizes cost and others that calculate the reliability indices, and not all variables are involved in both problems, the final result is that only some sensitivities are obtained. In addition, the method requires the use of a relaxation factor, that has to be fixed experimentally. In this paper, an alternative procedure that avoids the relaxation factor and allows to perform a complete sensitivity analysis is presented.

The remaining of this paper is structured as follows. In Section 2 the FPSF method for designing engineering works is presented and the methods proposed by Castillo et al. [7, 8] for performing a sensitivity analysis are reviewed. In Section 3 an alternative method optimized to perform a complete sensitivity analysis is presented. In Sections 4 and 5, examples of a breakwater and a bridge girder are given to illustrate the new proposals. Finally, in Section 6 some conclusions are drawn.

2 The failure probability-safety factor design method

It is important and clarifying to classify the set of variables involved in an engineering design problem into the following four subsets:

- $\boldsymbol{d}$: *Optimization design variables.* They are the design variables which values are to be chosen by the optimization program to optimize the objective function (minimize the cost). Normally, they define the dimensions of the work being design, as width, thickness, height, cross sections, etc.

- $\boldsymbol{\eta}$: *Non-optimization design variables.* They are the set of variables which mean or characteristic values are fixed by the engineer or the code and must be given as data to the optimization program. Some examples are costs, material properties (unit weights, strength, Young modula, etc.), and other geometric dimensions of the work being designed.

- $\boldsymbol{\kappa}$: *Random model parameters.* They are the set of parameters defining the random variability and dependence structure of the variables involved. For example, standard deviations, correlation coefficients, etc.

- $\boldsymbol{\psi}$: *Auxiliary or non-basic variables.* They are auxiliary variables which values can be obtained from the basic variables $\boldsymbol{d}$ and $\boldsymbol{\eta}$, using some formulas. They are used to facilitate the calculations and the statement of the problem constraints.

Examples of this classification are later given for the breakwater and the bridge crane examples.

Then, the engineering design problem (see Castillo et al. [7, 8]) can be stated as:

$$\underset{\bar{\boldsymbol{d}}}{\text{Minimize}} \quad c(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \tag{1}$$

subject to

$$g_i(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \geq F_i^0; \ \forall i \in I \tag{2}$$

$$\beta_i(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) \geq \beta_i^0; \ \forall i \in I \tag{3}$$

$$h(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \boldsymbol{\psi} \tag{4}$$

$$r_j(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \leq 0; \ \forall j \in J \tag{5}$$

where the bars and tildes refer to mean or characteristic values of the variables, $c(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})$ is the objective function to be optimized (cost function), (2) are the limit state equations related to the different failure modes, (3) are constraints that fix the lower bounds on the reliability indices, (4) are the equations that allow obtaining the auxiliary variables $\boldsymbol{\psi}$ from the basic variables $\boldsymbol{d}$ and $\boldsymbol{\eta}$, and (5) are the geometric or code constraints.

Unfortunately, this problem cannot be solved directly because each of the constraints (3) involve a complicated integral or another optimization problem, i.e.,

$$\beta_i(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) = \underset{\boldsymbol{d}_i, \boldsymbol{\eta}_i}{\text{Minimum}} \quad \beta_i = \sqrt{\sum_{j=1}^{n} z_j^2} \tag{6}$$

subject to

$$\begin{aligned} g_i(\boldsymbol{d}_i, \boldsymbol{\eta}_i) &= 1 &(7)\\ T(\boldsymbol{d}_i, \boldsymbol{\eta}_i; \bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) &= \mathbf{z} &(8)\\ h(\boldsymbol{d}_i, \boldsymbol{\eta}_i) &= \boldsymbol{\psi} &(9) \end{aligned}$$

where $\boldsymbol{d}_i$ and $\boldsymbol{\eta}_i$ are the design points associated with the design $\boldsymbol{d}$ and $\boldsymbol{\eta}$ random variables for failure mode i, and $T(\boldsymbol{d}_i, \boldsymbol{\eta}_i; \bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa})$ is the usual transformation (Rosenblatt, Nataf) that converts $\boldsymbol{d}_i$ and $\boldsymbol{\eta}_i$ into the standard independent normal random variables $\mathbf{z}$.

The method suggested by Castillo et al. [7, 8] for solving this problem proceeds by iterations that consist of repeating a sequence of three steps: (1) an optimal (in the sense of optimizing an objective function) classic design, based on given safety factors, is done, (2) failure probabilities (reliability indices) or bounds against all failures modes are determined, and (3) all mode safety factor bounds are adjusted using a relaxation factor. The three steps are repeated until convergence, i.e., until the safety factors lower bounds and the mode of failure probability upper bounds are satisfied. More precisely, the method proceeds as follows:

Step 1. Solving the optimal classic design. An optimal classic design based on the actual safety factors, that are fixed initially to their corresponding lower bounds, is done. In other words, the following problem is solved (both the standard and the sensitivity analysis oriented statements of this problem are provided):

Standard statement

$$\underset{\bar{\boldsymbol{d}}}{\text{Minimize}} \quad c\;(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})$$

subject to

$$\begin{aligned} g_i(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) &\geq F_i^0;\ \forall i \in I\\ h(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) &= \boldsymbol{\psi}\\ r_j(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) &\leq 0;\ \forall j \in J \end{aligned}$$

Sensitivity analysis statement

$$\underset{\bar{\boldsymbol{d}}, \boldsymbol{\eta}}{\text{Minimize}} \quad c\;(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) \tag{10}$$

subject to

$$\begin{aligned} g_i(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &\geq F_i^0;\ \forall i \in I\\ h(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &= \boldsymbol{\psi}\\ r_j(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &\leq 0;\ \forall j \in J\\ \boldsymbol{\eta} &= \tilde{\boldsymbol{\eta}} \end{aligned} \tag{11}$$

The result of this process is a set of values of the design variables that satisfy the safety factor, the auxiliary, the geometric and the code constraints (11). The right hand side problem allows determining the sensitivities of the cost function $c(\bar{\boldsymbol{d}}, \boldsymbol{\eta})$ with respect to the $\tilde{\boldsymbol{\eta}}$ values (they are the values of the dual variables associated with the last constraints). Sensitivities with respect to F_i^0 can be obtained from both problems.

Step 2. Evaluating new β-values. The actual β-values associated with all modes of failure are evaluated, based on the design values of Step 1, solving for any $i \in I$ the problem:

<u>**Standard statement**</u>	<u>**Sensitivity analysis statement**</u>
$\underset{\boldsymbol{d}_i, \boldsymbol{\eta}_i}{\text{Minimize}} \quad \beta_i = \sqrt{\sum_{j=1}^{n} z_j^2}$	$\underset{\boldsymbol{d}_i, \boldsymbol{\eta}_i, \boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\kappa}_{\text{aux}}}{\text{Minimize}} \quad \beta_i = \sqrt{\sum_{j=1}^{n} z_j^2} \qquad (12)$

subject to

$$
\begin{array}{rcl}
g_i(\boldsymbol{d}_i, \boldsymbol{\eta}_i) & = & 1 \\
T(\boldsymbol{d}_i, \boldsymbol{\eta}_i; \bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}) & = & \mathbf{z} \\
h(\boldsymbol{d}_i, \boldsymbol{\eta}_i) & = & \boldsymbol{\psi}
\end{array}
\qquad
\begin{array}{rcl}
g_i(\boldsymbol{d}_i, \boldsymbol{\eta}_i) & = & 1 \\
T(\boldsymbol{d}_i, \boldsymbol{\eta}_i; \bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\phi}) & = & \mathbf{z} \\
h(\boldsymbol{d}_i, \boldsymbol{\eta}_i) & = & \boldsymbol{\psi} \\
\boldsymbol{d} & = & \bar{\boldsymbol{d}} \\
\boldsymbol{\eta} & = & \tilde{\boldsymbol{\eta}} \\
\boldsymbol{\kappa}_{\text{aux}} & = & \boldsymbol{\kappa}
\end{array}
\qquad (13)
$$

At this step as many optimization problems as the number of modes of failure are solved, and the sensitivities of the reliability indices β_i with respect to $\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$ can be obtained from the right hand side problem.

Additionally, the design points or points of maximum likelihood $\boldsymbol{d}_i^*$ and $\boldsymbol{\eta}_i^*$ for each mode of failure I are obtained (optimal solutions of the problem (12)-(13)).

Step 3: Updating safety factor values. The safety factors bounds are adequately updated for the actual safety factors and the actual β-values to satisfy the required bounds. To this end, the safety factors are modified using the expression

$$F_i = \max\left(F_i + \rho\ (\beta_i^0 - \beta_i), F_i^0\right); \quad i \in I, \qquad (14)$$

where ρ is a small positive constant (relaxation factor). The max function is used in order to guarantee that the classical safety factors constraints are satisfied.

3 Alternative solution method

The iterative method presented in Section 2 has two shortcomings:

1. It requires a relaxation factor ρ that need to be fixed by trial and error. An adequate selection leads to a fast convergence of the process, but an inadequate selection can lead to lack of convergence.

2. The cost sensitivities with respect to F_i^0 and $\tilde{\boldsymbol{\eta}}$, and the β-sensitivities with respect to $\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}, \boldsymbol{\kappa}$ can be directly determined, but the cost sensitivities with respect to β_i^0 and $\boldsymbol{\kappa}$ are not easily available.

In this section an alternative method is given that solves both shortcomings, and in addition exhibit a better convergence. The method is as follows.

$$\textbf{Master Problem:} \quad \underset{\bar{\boldsymbol{d}}, \boldsymbol{\eta}}{\text{Minimize}} \quad c(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) \qquad (15)$$

subject to

$$
\begin{aligned}
g_i(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &\geq F_i^0;\ i \in I && (16)\\
\beta_i^{(k)} + \boldsymbol{\lambda}_i^{(k)^T}(\bar{\boldsymbol{d}} - \bar{\boldsymbol{d}}^{(k)}) + \boldsymbol{\mu}_i^{(k)^T}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + \boldsymbol{\delta}_i^{(k)^T}(\boldsymbol{\kappa}_{\text{aux}} - \boldsymbol{\kappa}) &\geq \beta_i^0;\ i \in I; k = 1, \cdots, \nu - 1 && (17)\\
h(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &= \boldsymbol{\psi} && (18)\\
r_j(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) &\leq 0;\ \forall j \in J && (19)\\
\boldsymbol{\eta} &= \tilde{\boldsymbol{\eta}} && (20)\\
\boldsymbol{\kappa}_{\text{aux}} &= \boldsymbol{\kappa} && (21)
\end{aligned}
$$

where we obtain $\bar{\boldsymbol{d}}^{(\nu)}$, and $\forall i$:

$$
\textbf{Subproblem: } \beta_i^{(\nu)} = \underset{\boldsymbol{d}_i, \boldsymbol{\eta}_i, \mathbf{z}, \boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\kappa}_{\text{aux}}}{\text{Minimize}} \quad \beta_i = \sqrt{\sum_{j=1}^{n} z_j^2} \qquad (22)
$$

subject to

$$
\begin{aligned}
g_i(\boldsymbol{d}_i, \boldsymbol{\eta}_i) &= 1 && (23)\\
T(\boldsymbol{d}_i, \boldsymbol{\eta}_i; \boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{\kappa}) &= \mathbf{z} && (24)\\
h(\boldsymbol{d}_i, \boldsymbol{\eta}_i) &= \boldsymbol{\psi} && (25)\\
\boldsymbol{d} &= \bar{\boldsymbol{d}}^{(\nu)} : \boldsymbol{\lambda}_i^{(\nu)} && (26)\\
\boldsymbol{\eta} &= \tilde{\boldsymbol{\eta}} : \boldsymbol{\mu}_i^{(\nu)} && (27)\\
\boldsymbol{\kappa}_{\text{aux}} &= \boldsymbol{\kappa} : \boldsymbol{\delta}_i^{(\nu)} && (28)
\end{aligned}
$$

The process of solving iteratively these two problems is repeated starting from $\nu = 0$ and increasing the value of ν in one unit, until convergence of their solutions is obtained. Note that at iteration $\nu = 0$ there is no hyperplane approximation (17) of constraint (3).

It should be noted that problem (15)-(28) is a relaxation of problem (1)-(5) in the sense that functions $\beta_i(\cdot)$ are approximated using cutting hyperplanes. Functions $\beta_i(\cdot)$ become more precisely approximated as the iterative procedure progresses, which implies that problem (15)-(28) reproduces more exactly problem (1)-(5) (see Kelly [13]). Observe, additionally, that cutting hyperplanes are constructed using the dual variable vector associated with constraints (26), (27) and (28) in problems (22)-(28) (the subproblems).

It should be noted that equations (26), (27) and (28) allow determining the sensitivities of the reliability indices β_i with respect to $\bar{\mathbf{d}}$, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\kappa}$ in a straightforward manner. Similarly, equations (17) and (21) allow obtaining the sensitivities of the cost with respect to β_i^0 and $\boldsymbol{\kappa}$, which are the new contributions of the proposed method. Thus, in addition to a better convergence rate, the proposed method permits determining extra sensitivities.

From the computational point of view it is very important to mention that the solution obtained using the proposed method with or without sensitivity analysis is the same. Therefore, to improve the efficiency of the process the problems are solved without sensitivity analysis, and once the optimal solution is attained the same problem with sensitivity analysis is solved using just one iteration because the starting point is the optimal one. Thus, convergence in the sensitivity analysis is guaranteed.

3.1 The case of partial safety factors

An additional advantage of the proposed methodology is that partial safety factors associated with random variables can be used, instead of global safety factors. Thus, the method can be used as a calibration tool of the standard codes based on partial safety factors.

Then, the engineering design problem in this case can be stated as:

$$\underset{\bar{\boldsymbol{d}}}{\text{Minimize}} \quad c(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) \tag{29}$$

subject to

$$g_i(\boldsymbol{\gamma}^T(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})) \geq 1; \ \forall i \in I \tag{30}$$

$$\beta_i(\boldsymbol{\gamma}^T(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}), \boldsymbol{\kappa}) \geq \beta_i^0; \ \forall i \in I \tag{31}$$

$$h(\boldsymbol{\gamma}^T(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})) = \boldsymbol{\psi} \tag{32}$$

$$r_j(\boldsymbol{\gamma}^T(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})) \leq 0; \ \forall j \in J \tag{33}$$

where $\boldsymbol{\gamma}$ is the vector of partial safety factors related to variables $\bar{\boldsymbol{d}}$ and $\tilde{\boldsymbol{\eta}}$, and T is used to refer to the transpose matrix.

In this case the proposed method proceeds as follows:

$$\textbf{Master Problem:} \quad \underset{\bar{\boldsymbol{d}}, \boldsymbol{\eta}, \gamma_{\text{aux}}}{\text{Minimize}} \quad c(\bar{\boldsymbol{d}}, \boldsymbol{\eta}) \tag{34}$$

subject to

$$g_i(\boldsymbol{\gamma}_{\text{aux}}^T(\bar{\boldsymbol{d}}, \boldsymbol{\eta})) \geq 1; \ \ i \in I \tag{35}$$

$$\beta_i^{(k)} + \boldsymbol{\lambda}_i^{(k)^T}(\bar{\boldsymbol{d}} - \bar{\boldsymbol{d}}^{(k)}) + \boldsymbol{\mu}_i^{(k)^T}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + \boldsymbol{\delta}_i^{(k)^T}(\boldsymbol{\kappa}_{\text{aux}} - \boldsymbol{\kappa}) \geq \beta_i^0; \ i \in I; k = 1, \cdots, \nu - 1 \tag{36}$$

$$h(\boldsymbol{\gamma}_{\text{aux}}^T(\bar{\boldsymbol{d}}, \boldsymbol{\eta})) = \boldsymbol{\psi} \tag{37}$$

$$r_j(\boldsymbol{\gamma}_{\text{aux}}^T(\bar{\boldsymbol{d}}, \boldsymbol{\eta})) \leq 0; \ \forall j \in J \tag{38}$$

$$\boldsymbol{\eta} = \tilde{\boldsymbol{\eta}} \tag{39}$$

$$\boldsymbol{\kappa}_{\text{aux}} = \boldsymbol{\kappa} \tag{40}$$

$$\gamma_{\text{aux}} = \boldsymbol{\gamma} \tag{41}$$

where we obtain $\bar{\boldsymbol{d}}^{(\nu)}$, then the same subproblem (22)-(28) is solved for all failures modes. The process of solving iteratively these two problems is repeated starting from $\nu = 0$ and increasing the value of ν in one unit, until convergence of their solutions is obtained.

Note that using this method is possible to obtain the cost sensitivities with respect the partial safety factors $\boldsymbol{\gamma}$.

4 Example of application. Design of a rubblemound breakwater

Consider the construction of a rubblemound breakwater (see Figure 1) to protect a harbor area from high waves during a storm. The breakwater must be strong enough to survive the attack of storm waves, and the crest must be high enough to prevent the intrusion of sea water onto the harbor by overtopping. For simplicity, only overtopping failure is considered. Other failure modes, such us armor failure, crownwall sliding failure, etc. are ignored.

Our goal is an optimal design of the breakwater based on minimizing the construction cost C_{co}:

$$C_{\mathrm{co}} = c_{\mathrm{c}} v_{\mathrm{c}} + c_{\mathrm{a}} v_{\mathrm{a}}$$

where v_c and v_a are the concrete and armor volumes, respectively, and c_c and c_a are the respective construction costs per unit volume.

For a rubblemound breakwater of slope $\tan \alpha_{\mathrm{s}}$ and freeboard F_{c} (see Figure 1), and a given wave of height H and period T, the volume of water that overtops the structure can be estimated from the volume of water that would rise over the extension of the slope exceeding the freeboard level. With this approximation, overtopping (failure) occurs whenever the difference between the maximum excursion of water over the slope, R_{u}, called wave run-up, exceeds the freeboard F_{c}. Thus, overtopping failure occurs if

$$F_{\mathrm{c}} - R_{\mathrm{u}} < 0 \tag{42}$$

Based on experiments, the following equation has been proposed to evaluate the dimensionless quantity R_{u}/H:

$$\frac{R_{\mathrm{u}}}{H} = A_{\mathrm{u}} \left(1 - e^{B_{\mathrm{u}} I_{\mathrm{r}}}\right)$$

where A_{u} and B_{u} are given coefficients that depend on the armor units and I_{r} is the Iribarren number

$$I_{\mathrm{r}} = \frac{\tan \alpha_{\mathrm{s}}}{\sqrt{H/L}}$$

where α_s is the seaside slope angle and L is the wave length, obtained from the dispersion equation

$$\left(\frac{2\pi}{T}\right)^2 = g\frac{2\pi}{L} \tanh \frac{2\pi D_{\mathrm{wl}}}{L}$$

and D_{wl} is the design water level.

In addition, due to construction reasons the slope α_{s} is limited by:

$$1/3 \leq \tan \alpha_{\mathrm{s}} \leq 2/3$$

The set of variables and parameters involved in this problem, as indicated in Section 2, can be partitioned into four subsets:

1. *Optimization design variables*: $\boldsymbol{d} = \{F_{\mathrm{c}}, \tan \alpha_{\mathrm{s}}\}$.

2. *Non-optimization design variables*: $\boldsymbol{\eta} = \{A_{\mathrm{u}}, B_{\mathrm{u}}, D_{\mathrm{wl}}, H, T, c_{\mathrm{c}}, c_{\mathrm{a}}\}$.

3. *Random model parameters*: $\boldsymbol{\kappa} = \{H_{\mathrm{s}}, \bar{T}, d_{\mathrm{st}}, \sigma_{A_u}, \sigma_{B_u}\}$, where H_{s} is the significant wave height, $\bar{T}$ is the average value of the period of the sea waves, and d_{st} is the sea state duration.

4. *Auxiliary or non-basic variables*: $\boldsymbol{\psi} = \{I_{\mathrm{r}}, v_{\mathrm{a}}, v_{\mathrm{c}}, R_{\mathrm{u}}, L, d\}$.

The basic random variables in this problem are H, T, A_u and B_u. All variables are assumed to be independent, H and T with cumulative distribution functions:

$$F_H(H) = 1 - e^{-2(H/H_\mathrm{s})^2}; \quad H \geq 0 \tag{43}$$

and

$$F_T(T) = 1 - e^{-0.675(T/\bar{T})^4}; \quad T \geq 0 \tag{44}$$

and A_u and B_u are assumed to be normal $N(\mu_{A_u}, \sigma_{A_u})$ and $N(\mu_{B_u}, \sigma_{B_u})$, respectively.

If P_f is the probability of overtopping failure due to a single wave, the lifetime breakwater failure probability becomes

$$P_\mathrm{f}^\mathrm{D}(\boldsymbol{d}) = 1 - (1 - P_\mathrm{f}(\boldsymbol{d}))^N \tag{45}$$

where $N = \theta d_\mathrm{st}/\bar{T}$ is the equivalent number of waves during the design sea state for period D, θ is a coefficient measuring the degree of independence of the waves ($\theta = 1$ for independence and $\theta = 0$ for complete dependence), d_st is its duration, and $\bar{T}$ is the mean period of waves. Equation (45) leads to

$$\Phi(-\beta^0) = P_\mathrm{f}(\boldsymbol{d}) = 1 - \left(1 - P_\mathrm{f}^\mathrm{D}(\boldsymbol{d})\right)^{1/N} \tag{46}$$

where β^0 is the reliability index associated with a single wave.

Then, the design problem consists of solving the following problem, equivalent to (1)-(5):

$$\underset{F_c, \tan\alpha_s}{\text{Minimize}} \quad C_\mathrm{co} = c_c v_c + c_a v_a \tag{47}$$

subject to

$$F_c/R_u \geq F^0 \tag{48}$$

$$\beta \geq \beta^0 \tag{49}$$

$$\frac{R_u}{\tilde{H}} = \mu_{A_u}\left(1 - e^{\mu_{B_u} I_r}\right) \tag{50}$$

$$I_r = \frac{\tan\alpha_s}{\sqrt{\tilde{H}/L}} \tag{51}$$

$$\left(\frac{2\pi}{\tilde{T}}\right)^2 = g\frac{2\pi}{L}\tanh\frac{2\pi D_\mathrm{wl}}{L} \tag{52}$$

$$v_c = 10d \tag{53}$$

$$v_a = \frac{1}{2}(D_{WL} + 2)(46 + D_{WL} + \frac{(D_{WL} + 2)}{\tan\alpha_s^{(\nu)}}) \tag{54}$$

$$F_c = 2 + d \tag{55}$$

$$1/3 \leq \tan\alpha_\mathrm{s} \leq 2/3 \tag{56}$$

where $\tilde{H} = 1.8H_s$ and $\tilde{T} = 1.1T_z$ are the characteristic values of H and T, respectively, that are used as deterministic values in the classical design. Note that equation (48) corresponds to (2), (49) is (3), (50)-(55) are associated with (4) and (56) corresponds to (5).

The reliability index against overtopping β is obtained solving the following problem:

$$\underset{H, T, A_u, B_u}{\text{Minimize}} \quad \beta = \sqrt{\sum_{j=1}^{4} z_j^2} \tag{57}$$

$$\begin{aligned}
\Phi(z_1) &= 1 - e^{-2(H/H_s)^2} && (58)\\
\Phi(z_2) &= 1 - e^{-0.675(T/\bar{T})^4} && (59)\\
z_3 &= \frac{A_u - \mu_{A_u}}{\sigma_{A_u}} && (60)\\
z_4 &= \frac{B_u - \mu_{B_u}}{\sigma_{B_u}} && (61)\\
F_c/R_u &= 1 && (62)\\
\frac{R_u}{H} &= A_u\left(1 - e^{B_u I_r}\right) && (63)\\
I_r &= \frac{\tan\alpha_s}{\sqrt{H/L}} && (64)\\
\left(\frac{2\pi}{T}\right)^2 &= g\frac{2\pi}{L}\tanh\frac{2\pi D_{\text{wl}}}{L} && (65)
\end{aligned}$$

where (58)-(61) correspond to the Rosenblatt transformation (8), (62) correspond to (7) and (63)-(65) correspond to (9).

4.1 A numerical example

To perform a reliability-based design of a particular rubblemound breakwater, assume the following values for the variables involved:

$$D_{wl} = 20\ m;\ A_u \sim N(1.05, 0.3^2)\ ;\ B_u \sim N(-0.67, 0.134^2)\ ;\ c_c = 60\ euro/m^3;$$

$$c_a = 2.4\ euro/m^3;\ H_s = 5\ m;\ \bar{T} = 10\ s;\ d_{st} = 1\ h;\ D = 1 \text{ year};\ P_f^D = 0.5;\ F^0 = 1.05$$

This leads to (see (46)) $P_f = 0.00192$ and $\beta^0 = 2.89$.

The solution of this problem, using the method above, is shown in Table 1. The error column measures the difference between consecutive iterations. It turns out that convergence of the process requires only 3 iterations.

Note that since the initial design does not satisfies the reliability index constraint, the final design is more expensive than the initial one. This means that the reliability index constraint is more strict than the safety factor constraint. More precisely, the reliability index is $\beta = 2.89$, the same as the lower bound selected β^0, and the real safety factor is greater than the bound $F^0 = 1.05$.

A complete sensitivity analysis for the cost and the β sensitivities is shown in Table 2. Note that the cost increases and the reliabilities decrease as the dispersions ($\sigma_{A_u}, \sigma_{B_u}$) increase. Note also that an increase of D_{wl} increases the cost and decreases the reliability

Table 1: Illustration of the iterative process.

ν	$C_{\text{co}}^{(\nu)}$	$\tan\alpha_s^{(\nu)}$	$F_c^{(\nu)}$	$F^{(\nu)}$	$\beta^{(\nu)}$	$\text{error}^{(\nu)}$
0	5746.9	0.333	5.770	1.050	2.036	0.4193622
1	6817.3	0.333	7.554	1.375	2.826	0.2793755
2	6911.5	0.333	7.711	1.403	2.890	0.0221902
3	6912.0	0.333	7.712	1.403	$\underline{2.890}$	0.0001303

Table 2: Sensitivities for the rubblemound breakwater problem.

x	$\dfrac{\partial c(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}})}{\partial x}$	$\dfrac{\partial \beta}{\partial x}$
$\tan\alpha_s$	–	-4.854
F_c	–	0.406
μ_{A_u}	2949	-1.996
μ_{B_u}	-3073	2.080
H_s	686	-0.464
T_z	145	-0.098
σ_{A_u}	5103	-3.454
σ_{B_u}	2475	-1.675
D_{wl}	288	-0.016
c_c	57	0.000
c_a	1452	0.000
F^0	0	–
β^0	1477	–

against overtopping. This is a valuable information for code makers and construction engineers, because they can know the cost or reliability increase due to a change in safety factors, failure probability bounds, or uncertainty parameters. Note also that since the reliability index lower bound is active its cost sensitivity is 1477, whereas the cost sensitivity related to the safety factor lower bound is 0, because constraint (48) is inactive.

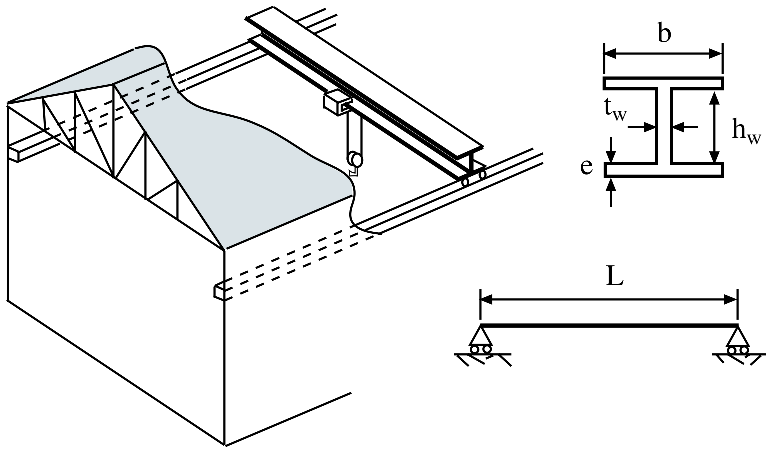

Figure 2: Illustration of the bridge crane.

5 Bridge Crane Design

Modern industries require equipment for handling large, heavy, or bulky objects. That is why there are engineers specializing in overhead material handling: Bridge Cranes, Hoists, and Monorails.

An under running overhead crane with single girder is shown in Figure 2. All its structural elements must be manufactured in accordance with current mandatory requirements of the National Safety and Health Act, OSHA Section 1910.179 and 1910.309 as applicable. Additionally, cranes must be manufactured in accordance with the appropriate standard of ANSI specifications, the National Electric Code, and the Crane Manufacturers Association of America (CMAA) specifications. Crane girders are designed and built using, structural steel beams (reinforced as necessary) or plate box sections. Bridge girder to end truck connections are designed for loadings, stresses and stability in accordance with current CMAA design specifications.

In this Section we apply the engineering design method developed in Section 3 to the design of an overhead crane (see Figure 2). In particular, the bridge girder dimensions that allow trolley travelling horizontally are calculated. It consists of a box section fabricated from plate of structural steel, for the web, top and bottom plates, so as to provide for maximum strength at minimum dead weight. Maximum allowable vertical girder deflection shall be a function of span.

Consider the girder and the cross section shown in Figure 2, where L is the span or distance from centerline to centerline of runway rails, b and e are the flange width and thikness, respectively and h_w and t_w are the web height and thikness, respectively.

As indicated in Section 2, the set of variables involved in the problem can be partitioned into four subsets:

1. *Optimization design variables*: $\boldsymbol{d} = \{b, e, t_w, h_w\}$.

2. *Non-optimization design variables*: $\boldsymbol{\eta} = \{P, f_y, E, \nu, \gamma_y, L, c_y\}$, where P is the maximum load supported by the girder, f_y is the value of the elastic limit of structural

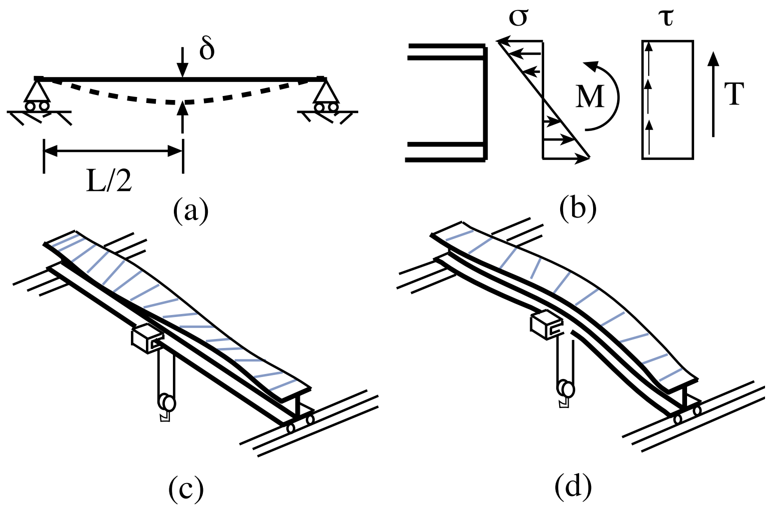

Figure 3: Illustration of the bridge girder modes of failure.

steel, E is the Young modulus of the steel, ν is the Poisson modulus, γ_y is the steel unit weight, L is the span length and c_y is the steel cost.

3. *Random model parameters*: $\boldsymbol{\kappa} = \{\lambda_P, \delta_P, \sigma_{f_y}, \sigma_E, \sigma_\nu, cv_L, cv_{\gamma_y}, cv_b, cv_e, cv_{t_w}, cv_{h_w}\}$, where cv refers to the coefficient of variation and σ to standard deviation of the corresponding variable, λ_P and δ_P are the Gumbel distribution model parameters for the maximum load and cv_d is the coefficient of variation of the optimization design variables.

4. *Auxiliary or non-basic variables*: (to be defined later)

$$\boldsymbol{\psi} = \{W, I_{xx}, I_{yy}, I_t, G, \sigma, \tau, M_{cr}, \delta, M, T\}.$$

In the classical approach the partial safety factors are used and the variables are assumed to be deterministic, i.e., the mean or characteristic (extreme percentiles) values of the variables are considered.

Assume that the following four failure modes are considered (see Figure 3):

1. *Maximum allowed deflection.* The maximum deflection constraint is defined (see Figure 3(a)) as the ratio

$$g_d(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \frac{\delta_{max}}{\delta} \tag{66}$$

where δ_{max} is the maximum deflection allowed by codes and δ is the maximum deflection at the center of the girder.

2. *Damage limit state of the steel upper and lower flanges.* The ratio of the actual strength to actual stresses

$$g_u(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \frac{f_y}{\sqrt{\sigma^2 + 3\tau^2}} \tag{67}$$

is the limit state constraint, and σ and τ are the normal and tangential stresses at the center of the beam, respectively.

3. *Damage limit state of the steel web.* The bearing capacity limit state is the ratio of the shear strength capacity to actual shear stress at the center of the beam

$$g_w(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \frac{f_y}{\sqrt{3}\tau} \tag{68}$$

4. *Global Buckling.* The global buckling limit state equation is the ratio of the critical moment against buckling of the cross section to the maximum moment applied at the center of the beam

$$g_d(\bar{\boldsymbol{d}}, \tilde{\boldsymbol{\eta}}) = \frac{M_{cr}}{M} \tag{69}$$

The gilder bridge is safe if and only if g_d, g_u, g_w and $g_b \geq 1$.

5.1 Design constraints

The following constraints are considered.

1. *Geometrical and mechanical properties of the girder.* The moments of inertia I_{xx} and I_{yy} are

$$I_{xx} = \frac{1}{12}\left(b(h_w + 2e)^3 - (b - t_w)h_w^3\right) \tag{70}$$

$$I_{yy} = \frac{1}{12}\left(2eb^3 - t_w h_w^3\right) \tag{71}$$

whereas the torsional moment of inertia is obtained using

$$I_t = \frac{1}{3}\left(2be^3 - h_w t_w^3\right) \tag{72}$$

The deflection at the center of the beam is calculated using:

$$\delta = \frac{PL^3}{48EI_{xx}} + \frac{5WL^4}{384EI_{xx}} \tag{73}$$

where W is the girder bridge weight per unit length

$$W = \gamma_s(2eb + t_w h_w) \tag{74}$$

The stresses at the center of the beam are calculated considering:

$$T = P/2 \tag{75}$$

$$M = PL/4 \tag{76}$$

where T and M are the shear force and moment, respectively. Thus,

$$\sigma = \frac{M(h_w + e)}{2I_{xx}} \quad (77)$$

$$\tau = \frac{T}{h_w t_w} \quad (78)$$

The critical moment for global buckling is

$$M_{cr} = \frac{\pi}{L}\sqrt{EGI_{yy}I_t} \quad (79)$$

with the auxiliary parameter

$$G = \frac{E}{2(1+\nu)}$$

2. *Code and other constraints.* The following constraints are fixed by the codes.

   The steel thickness must satisfy

$$0.008 \leq e \leq 0.038 \quad (80)$$

$$0.008 \leq t_w \leq 0.038 \quad (81)$$

   and the maximum deflection allowed is

$$\delta_{max} = L/888$$

   To avoid local buckling (see Figure 3(c)) the design satisfy the following restriction

$$\frac{b}{2e} \leq 15\sqrt{\frac{276}{f_y}} \quad (82)$$

   where f_y is the steel strength in MPa.

   To support the trolley unit which travels on the bottom flange of the bridge girder and carries the hoist, the minimum flange width must be 0.30 m.

5.2 A numerical example

To perform a probabilistic design in the bridge girder example using the partial safety factor method, the joint probability density of all variables is required. All basic random variables are assumed to be independent. The statistical distributions of the variables involved are taken from the Probabilistic Model Code:

1. The maximum supported load has a Gumbel (Maximum) distribution with the following parameters

$$\lambda_P = 600\ kN; \quad \delta_P = 70.2\ kN$$

2. Variables f_y, E and ν are assumed to have log-normal distributions

$$\log f_y \sim N(\mu_{f_y}, \sigma_{f_y}); \;\; \log E \sim N(\mu_E, \sigma_E); \;\; \log \nu \sim N(\mu_\nu, \sigma_\nu)$$

where

$$\mu_{f_y} = \log(395\ MPa); \;\; \mu_E = \log(210000\ MPa); \;\; \mu_\nu = \log(0.3)$$

and the standard deviations:

$$\sigma_{f_y} = 0.07; \;\; \sigma_E = 0.03; \;\; \sigma_\nu = 0.03.$$

3. Variables L, γ_y, b, e, t_w and h_w have normal distributions

$$L \sim N(\mu_L, \mu_L cv_L); \;\; \gamma_y \sim N(\mu_{\gamma_y}, \mu_{\gamma_y} cv_{\gamma_y}); \;\; b \sim N(\mu_b, \mu_b cv_b)$$

$$e \sim N(\mu_e, \mu_e cv_e); \;\; t_w \sim N(\mu_{t_w}, \mu_{t_w} cv_{t_w}); \;\; h_w \sim N(\mu_{h_w}, \mu_{h_w} cv_{h_w})$$

The means of L and γ_y are:

$$\mu_L = 6\ m; \;\; \mu_{\gamma_y} = 78.5\ kN/m^3$$

and the means of the design variables b, e, t_w and h_w are the optimal values obtained from classical design and their coefficients of variation are:

$$cv_L = 0.01; \;\; cv_{\gamma_y} = 0.01; \;\; cv_b = cv_e = cv_{t_w} = cv_{h_w} = 0.01$$

The constant parameter is:

$$c_y = 0.24\ euro/kN$$

Assume also that the required reliability bounds are:

$$\beta_d^0 = 1.7; \;\; \beta_u^0 = 3.7; \;\; \beta_w^0 = 3.7; \;\; \beta_b^0 = 3.1$$

and the partial safety factors used in the classical problem are:

$$\gamma_P = 1.33; \;\; \gamma_{f_y} = 0.91; \;\; \gamma_E = 1.0; \;\; \gamma_\nu = 1.0; \;\; \gamma_{\gamma_y} = 1.0; \;\; \gamma_L = 1.0$$

Note that "violation" of limit states with more serious consequences are associated with higher reliability indices.

Using the Rosenblatt [18] transformation, this set is transformed into a set of standard independent normal $N(0,1)$ random variables $Z_1, Z_2, \cdots, Z_{10}$ by

$$\Phi(z_1) = \exp\left\{-\exp\left[-\frac{(P-\lambda_P)}{\delta_P}\right]\right\} \quad (83)$$

$$\begin{array}{llllll} z_2 = \dfrac{\log f_y - \mu_{f_y}}{\sigma_{f_y}} & z_3 = \dfrac{\log E - \mu_E}{\sigma_E} & z_4 = \dfrac{\log \nu - \mu_\nu}{\sigma_\nu} \\ z_5 = \dfrac{\gamma_y - \mu_{\gamma_y}}{\mu_{\gamma_y} cv_{\gamma_y}} & z_6 = \dfrac{L - \mu_L}{\mu_L cv_L} & z_7 = \dfrac{b - \mu_b}{\mu_b cv_d} \\ z_8 = \dfrac{e - \mu_e}{\mu_e cv_d} & z_9 = \dfrac{t_w - \mu_{t_w}}{\mu_{t_w} cv_d} & z_{10} = \dfrac{h_w - \mu_{h_w}}{\mu_{h_w} cv_d} \end{array} \quad (84)$$

Then, minimizing the construction cost (C_{co}), using the proposed method, the results shown in Table 3 are obtained. It shows the progress and convergence of the process, that is attained after 8 iterations. The initial ($\nu = 0$) iteration column shows the values of the design variables, and the actual failure β-values, associated with the optimal classical design. Note that the β_b and β_d constraints (underlined in Table 3) do not hold. Then, the iterative process continues until all constraints are satisfied.

The last column of the table shows the values of the design variables

$$b, e, t_w, h_w$$

together with the final β-values.

The active values appear underlined in the last column of Table 3, from which the following conclusions can be drawn.

1. The process converges in only 8 iterations.

2. The list of actual β-reliability indices is obtained.

3. Due to the strict constraints imposed by the serviceability limit state (maximum deflection), and global buckling constraints the probability bounds β_b and β_d are active.

4. The final design (iteration 8) is more expensive than the initial design (iteration 0), because this does not satisfy the β_b and β_d constraints.

The sensitivities for the composite beam example are given in Table 4 that gives the cost sensitivities associated with the optimal classical design. It allows to know how much a small change in a single data value changes the total cost of the composite beam. This information is extremely useful during the construction process to control the cost, and for analyzing how the changes in the safety factors required by the codes influence the total cost of engineering works. For example, a change of one euro in the unit cost c_y of the steel leads to a cost increase of 10070 euros (see the corresponding entry in Table 4). Similarly, an increase in the partial safety factor γ_P does not change the cost (because the associated constraint is inactive), and an increase of one unit in the bridge span leads to an increase of the cost of 757 euros.

It also gives the sensitivities associated with the β-values too. It is useful to know how much a small change in a single data value changes the corresponding β-values, for example, the means, variation coefficients, etc. In this table the designer can easily analyze how the quality of the material (reduced standard deviations in f_y) or precision in the applied loads (reduced standard deviations in P) influence the safety of the beam. Note that an increase in the dispersion (standard deviations or coefficients of variation) leads to a decrease of the reliability β indices.

6 Conclusions

The main conclusions that can be derived from the previous sections are:

Table 3: Illustration of the iterative process.

ν	Units	0	1	2	3	4	5	6	7	8
$C_{co}^{(\nu)}$	*euro*	2304.1	2298.3	2464.6	2524.3	2549.8	2558.3	2560.2	2560.8	2560.9
$b^{(\nu)}$	*cm*	43.14	30.00	37.89	33.52	36.35	35.05	35.75	35.41	35.59
$e^{(\nu)}$	*mm*	16.41	24.20	21.04	24.65	23.03	24.00	23.55	23.79	23.67
$t_w^{(\nu)}$	*mm*	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
$h_w^{(\nu)}$	*cm*	77.83	72.67	73.24	72.64	72.68	72.57	72.62	72.59	72.61
$\beta_u^{(\nu)}$	–	4.127	3.873	4.170	4.226	4.265	4.271	4.276	4.276	4.277
$\beta_t^{(\nu)}$	–	7.161	6.836	6.872	6.834	6.837	6.830	6.833	6.831	6.832
$\beta_b^{(\nu)}$	–	$\underline{1.547}$	1.666	2.622	2.916	3.049	3.087	3.097	3.099	$\underline{3.100}$
$\beta_d^{(\nu)}$	–	$\underline{1.549}$	0.955	1.493	1.623	1.673	1.693	1.698	1.700	$\underline{1.700}$

1. The failure-probability-safety-factor method for engineering design gives a dual information on the safety of the structures being designed: safety factors and failure probabilities, giving a double way of safety control, and interesting calibration possibilities for the classic and probability based designs. Errors in the safety factor assumptions approach can be detected by the failure probability approach and vice versa.

2. The proposed alternative method for solving the failure-probability-safety-factor method converges in a few iterations, has a robust computational behavior, and is faster than the method proposed in Castillo et al. [7, 8], that uses a relaxation factor.

3. Since the alternative proposed method involves all the adequate variables in the right hand side of the constraints, a complete sensitivity analysis of the cost function and the β reliabilities associated with all modes of failure with respect to all data values can be easily performed.

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Table 4: Sensitivities.

x	$\frac{\partial C_{\text{co}}}{\partial x}$	$\frac{\partial \beta_u}{\partial x}$	$\frac{\partial \beta_t}{\partial x}$	$\frac{\partial \beta_b}{\partial x}$	$\frac{\partial \beta_d}{\partial x}$
e	–	7.472	0.000	25.26	13.55
b	–	111.9	0.000	372.5	216.8
t_w	–	202.2	586.3	32.74	63.97
h_w	–	6.300	6.460	0.120	15.09
λ_P	1.077	−0.003	−0.002	−0.004	−0.007
μ_{f_y}	0.001	0.000	0.000	0.000	0.000
μ_E	0.000	0.000	0.000	0.000	0.000
μ_ν	-272.5	0.000	0.000	1.656	0.000
μ_{γ_y}	30.82	0.000	0.000	0.000	−0.001
μ_L	757.3	−0.491	0.000	−1.513	−1.800
δ_P	5.886	−0.034	−0.049	−0.029	−0.020
σ_{f_y}	0.000	−5.476	−10.52	0.000	0.000
σ_E	0.201	0.000	0.000	−0.001	0.000
σ_ν	3.776	0.000	0.000	−0.023	0.000
cv_{γ_y}	0.002	0.000	0.000	0.000	0.000
cv_L	533.7	−0.370	0.000	−2.539	−1.975
cv_d	983.3	−1.614	−3.026	−4.943	−2.897
c_y	10070	0.000	0.000	0.000	0.000
γ_P	0.000	–	–	–	–
γ_{f_y}	265.0	–	–	–	–
γ_E	0.000	–	–	–	–
γ_ν	0.000	–	–	–	–
γ_{γ_y}	2417	–	–	–	–
γ_L	2417	–	–	–	–
β_u	0.000	–	–	–	–
β_w	0.000	–	–	–	–
β_b	164.6	–	–	–	–
β_d	58.61	–	–	–	–

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